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The Basis for a Room Global Temperature

M. G. Davies

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The basis for a room global temperature

By M. G. Davies

School of Architecture and Building Engineering, The University, Liverpool L69 3BX, U.K.

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The design of heating and cooling appliances in buildings in routine cases normally proceeds on the assumption of a room index temperature which combines the separate effects of air temperature and of the longwave radiant field in the enclosure. It is pointed out that the basis for the index in current use in the U.K. and elsewhere is flawed, and this article is concerned with the logic of setting up a valid index temperature in its place. The argument depends first on reducing the surface-to-surface radiant exchange between enclosure surfaces to an approximately equivalent surface-to-star point exchange, using a least-squares fit. The fit proves to be quite good. It is next established that to a limited extent the star point temperature – a fictitious construct – will do duty for the space-averaged observable radiant temperature in the room. Thirdly, since the index temperature is taken to drive the radiant and convective heat flows from the room as a whole to one of its bounding surfaces, the question is discussed as to how reliably these physically dissimilar mechanisms can be formally merged in this way. Finally, simple expressions are given for enclosure heat needs in relation to comfort temperature and similar quantities. The arguments present some innovative features in building heat transfer.

Notation

(a) Heat inputs

- Q_t combined radiant and convective input from an internal source (W)
- Q_r radiant component from an internal source (W)
- Q_a convective component from an internal source (W)
- p Q_r/Q_t
- Q_j input at surface j due to conduction, convection and surface heating (W)
- Q_{rj} radiant flux falling on surface j and taken to be absorbed at T_{jb} (W)
- Q_{fj} heat loss by conduction from T_j to T_o (W)
- Q_v heat loss by infiltration from T_{av} to T_o (W)

(b) Temperatures

- T_{ap} local or point air temperature (°C)
- T_{av} volume-averaged air temperature (°C)
- T_{rp} local observable radiant temperature (°C or K)
- T_{rv} volume-averaged observable radiant temperature (°C or K)
- β_{rv} non-dimensional version of T_{rv} due to an internal radiant source
- γ_{rv} non-dimensional version of T_{rv} due to a non-zero surface temperature
- T_{rs} radiant star temperature, taken to be an estimate of T_{rv} (°C or K)

β_{rs} non-dimensional version of T_{rs} due to an internal radiant source

γ_{rs} non-dimensional version of T_{rs} due to a non-zero surface temperature

T_{ra} rad-air temperature, composed of T_{av} and T_{rs} (°C)

T_c dry resultant temperature, $\frac{1}{2}T_{av}+\frac{1}{2}T_{rv}$ or $\frac{1}{2}T_{av}+\frac{1}{2}T_{rs}$ (°C)

T_j area-averaged temperature of surface j (°C)

T_{jb} black-body equivalent temperature of surface j (°C)

T_o ambient (outdoor) temperature (°C)

T_{sa} sol-air temperature (°C)

(*c*) *Geometrical and physical parameters*

l length of a rectangular room (m)

d depth of a rectangular room (m)

h height of a rectangular room (m)

A_j area of surface j (m^2)

d_{ji} thickness of layer i in wall j (m)

λ_{ji} conductivity of layer i in wall j (W m^{-1} K^{-1})

h_{cj} convective coefficient at an inside enclosure surface (W m^{-2} K^{-1})

h_{co} convective coefficient at an outside surface (W m^{-2} K^{-1})

W_j radiosity of surface j (W m^{-2})

F_{jk} view factor from surface j to surface k

σ Stefan–Bolzmann constant, 5.67×10^{-8} (W m^{-2} K^{-4})

h_r radiative heat transfer coefficient, $4\sigma T_{rs}^3$ and about 5.7 indoors (W m^{-2}K^{-1})

b_j A_j/total surface area of a rectangular room

β_j $\dfrac{\text{radiant conductance between } T_{jb} \text{ and an enveloping black-body surface}}{\text{radiant conductance between } T_{jb} \text{ and the radiant star node } T_{rs}}$

estimated as $1-b_j-3.54(b_j^2-0.5b_j)+5.03(b_j^3-0.25b_j)$

ϵ_j emissivity of surface j

E_j $((1-\epsilon_j)/\epsilon_j+\beta_j)^{-1}$

f_j fabric transmittance between T_j and T_o $(\Sigma d_{ji}/\lambda_{ji}+1/h_{co})^{-1}$ (W m^{-2} K^{-1})

u_j overall transmittance (U value) between T_{ra} and T_o via
$T_j = [1/(E_j h_r+h_{cj})+1/f_j]^{-1}$ (W m^{-2} K^{-1})

(*d*) *Conductances*

G_{jk} direct geometrical radiant conductance between black-body surfaces j and k (m^2)

G_{rs} total geometrical radiant conductance between T_{rs} and all enclosure surfaces (m^2)

R_{jk}^{Δ} exact overall geometrical radiant resistance between surfaces j and k (other surfaces being adiabatic) (m^{-2})

R_{jk}^{*} corresponding approximate overall resistance using a star-based network (m^{-2})

C_j convective conductance between T_{av} and $T_j = A_j h_{cj}$ (W K^{-1})

S_j radiant conductance between T_{rs} and $T_j = A_j E_j h_r$ (W K^{-1})

S $=\Sigma S_j$ (W K^{-1})

C $=\Sigma C_j$ (W K^{-1})

α $=C/S$

X conductance between T_{ra} and $T_{av}=(1+\alpha)C$ (W K^{-1})

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F_j fabric conductance between T_j and $T_o = A_j f_j$ (W K^{-1})

L_j fabric loss conductance between T_{ra} and T_o via $T_j = A_j u_j = (1/(S_j + C_j) + 1/F_j)^{-1}$ (W K^{-1})

L total fabric loss conductance $= \Sigma L_j$ (W K^{-1})

V ventilation conductance between T_{av} and T_o = volume throughput of air (m^3 s^{-1}) × volumetric specific heat (1200 J m^{-3} K^{-1}) (W K^{-1})

1. Introduction

Heat is transferred to and from the air in a room by the process of convection to the bounding surfaces of a room and to furnishings and occupants, and in particular heat is transferred by convection from hot surfaces to the air. Cool air may infiltrate from outside to mix with the room air and be heated before being lost from the room and, conversely, a ducted hot or cold air supply mixes with the room air and is lost at room air temperature.

The mechanism of radiation performs a similar function. Heat is exchanged by radiation between the cool surfaces of a room and any hot surface within it or forming part of the bounding surfaces. Radiation will be lost through an aperture such as an open window, a process similar to that of infiltration but of less significance.

A measuring device such as a thermometer senses the air temperature, intercepts the radiant field and records a value intermediate between strict air temperature at that point and the value it would indicate due to radiation in the absence of air.

Some general measure of room temperature is needed in connection with the design of room heating and cooling systems: it may serve to express temperature from the point of view of human thermal comfort, or it may serve as the temperature which drives the heat flow by conduction through the building fabric to the exterior and also, in effect, drives the loss due to air infiltration.

The traditional measure of temperature, in use in the U.K. up to the 1960s, was a so-called air temperature, T'_a say. All heat was taken to be input at T'_a whether by convection or radiation, all heat was driven from the room both by conduction and ventilation by T'_a, and T'_a served as the measure for purposes of thermal comfort. It is termed here 'so-called', because T'_a, though often referred to simply as 'air temperature', did in fact make some sort of allowance, willy-nilly, for the radiant field: the heat flow conductance between T'_a and any room surface was always taken to consist of both radiant and convective components of roughly equal magnitudes, and clearly, 'air temperature', strictly interpreted, cannot drive radiation, nor can radiation be input at it. Despite these objections, T'_a was and indeed remains a generally useful, if rough and ready, concept.

During the 1960s, workers at the U.K. Building Research Station were concerned with the overheating experienced in many of the much-glazed buildings that were fashionable at the time. They set up a new index temperature, 'environmental temperature', T_{ei}, which was intended to take formal account of the radiant field, a feature absent in T'_a.

By using conventional values for the convective heat transfer coefficient of h_c = 3 W m^{-2} K^{-1}, the radiative coefficient h_r of 5.7 W m^{-2} K^{-1} and surface emissivities of 0.9, T_{ei} was related to the strict air temperature T_{ai} and the area-weighted mean surface temperature T_m as

$$T_{ei} = \tfrac{2}{3}T_m + \tfrac{1}{3}T_{ai}.$$

(The coefficients totalled unity, but were otherwise approximate.) The associated thermal model involved explicit mention of T_{ai}, and of a conductance of $4.5\,\Sigma A$ (units $\mathrm{W\,K^{-1}}$) between T_{ei} and T_{ai}, where ΣA denoted the total room surface area. Dry resultant temperature T_c, the measure of thermal comfort, was expressed as

$$T_c = \tfrac{1}{2}T_m + \tfrac{1}{2}T_{ai}$$

and its value was located on the $4.5\,\Sigma A$ conductance. A convective input of heat Q_a was taken to act at T_{ai} and an input of long-wave radiation Q_r was treated as an input of $\tfrac{3}{2}Q_r$ at T_{ei} together with a *withdrawal* of $\tfrac{1}{2}Q_r$ from T_{ai}. The heat loss by ventilation was expressed using a conductance V between T_{ai} and ambient at T_o (V is the product of the volume air flow rate ($\mathrm{m^3\,s^{-1}}$) and the volumetric specific heat of air $1200\ \mathrm{J\,m^{-3}\,K^{-1}}$). The heat loss by conduction to ambient was expressed as the conductance ΣAU acting between T_{ei} and T_o, where U denotes the conventional U value associated with area A.

This model, the 'environmental temperature model', was taken over for professional use by the Institution of Heating and Ventilating Engineers (now the Chartered Institution of Building Services Engineers (CIBSE)) in the update of its 1970 Guide (IHVE Guide 1971), mostly in §A5. Its treatment was substantially changed in the 1979 update (CIBS 1979) and incorporated with little further change in the hardback version of 1986 (CIBSE 1986).

Unfortunately, the fundamental principles upon which this model is based are invalid (Davies 1986, 1989). The arguments include a number of more or less independent errors which stem from the fact that although T_{ei} was defined in terms of mean surface temperature T_m, it was interpreted as though T_m denoted mean radiant temperature T_{ri}. These are in fact quite separate concepts: T_m derives from measurements made at the bounding surfaces of a room, while T_{ri}, like air temperature, derives from measurements made within the space of the enclosure. There are problems too over the status of dry resultant temperature, the measure needed for appraisal of the thermal comfort the enclosure provides.

Over several years I have evolved a series of developments leading to a room index temperature which validly combines the effect of air and radiant temperatures (see Davies 1979, 1983). Some matters considered a decade ago have faded in importance and improvements in the presentation of others have been made. This article aims to bring these developments together into a single, definitive document, and as such it may serve to support the methods to be promulgated in the forthcoming update of the CIBSE Guide.

The argument falls into four sections. (i) The transformation of the pattern of surface–surface radiant exchange between the surfaces of a room to a surface-star point form (§2). (ii) The relationship between this star point temperature and observable radiant temperatures in a room (§3). (iii) The merging of the radiative and convective processes in a room so as to form a room index temperature, and its properties (§4). (iv) Finally, working formulae are given to express heat need in terms of comfort temperature (§5).

2. Radiant exchange in an enclosure

This section presents the fundamentals of radiant exchange in an enclosure and goes on to show how the surface to surface pattern of exchange can be represented with good approximation by a pattern centred on a radiant star node.

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(a) *Assumptions for radiant exchange*

The standard assumptions that are made to discuss radiant interchange between the surfaces of an enclosure are the following.

1. Each surface in the enclosure is supposed to be isothermal. If the temperature varies significantly over say the floor of a room, it can be subdivided into smaller substantially isothermal areas.
2. Each surface is supposed to be grey, that is, there is negligible variation of emissivity with wavelength.
3. Emissivity is assumed to be equal to absorptivity, and each is equal to one minus reflectivity.
4. It is assumed that when radiation is incident upon a surface, the reflected fraction has a uniform angular distribution, i.e. the surface reflects diffusely.
5. It is similarly assumed that the radiation emitted by the surface has a uniform angular distribution.
6. Finally, it is assumed that the radiation incident upon a surface is distributed uniformly over it, so that there is no focusing.

(b) *The thermal circuit formulation*

Radiant exchange can be expressed elegantly in the notation of a thermal circuit, as was first shown by Oppenheim (1956). We consider an enclosure consisting of several surfaces that exchange radiation one with another. The heat flow to or from a particular surface, surface 1 for example, may be due to conduction from behind the surface, by convection from the air within the enclosure or from some local heating, for example by electrical heating. Suppose that the net sum of these mechanisms is denoted by Q_1 and that it is to be further transferred by radiation. Suppose that the radiant flux leaving surface j is W_j (the radiosity) with units of watts per square metre. The radiant flow from surface j falling on surface 1 is $W_j A_j F_{j1}$, which equals $W_j A_1 F_{1j}$, where F_{1j} is the view factor from surface 1 to surface j. (We note the two relations for view factors, the first, $\Sigma F_{1j} = 1$ expressing the idea that all radiation leaving surface 1 is intercepted by surrounding surfaces, and that $A_1 F_{1j} = A_j F_{j1}$ and denotes the geometrical conductance between surfaces 1 and j.) The total energy arriving at surface 1 is accordingly the sum of all such radiant terms, together with Q_1.

The radiation leaving the surface is by definition $A_1 W_1$ (expressible as $\Sigma A_1 F_{1k} W_1$) and it consists of the flow emitted from surface 1 due to its temperature, $A_1 \epsilon_1 \sigma T_1^4$, together with the fraction $(1-\epsilon_1)$ of the incident radiation which is reflected from the surface.

Heat balance at the surface accordingly requires that

$$Q_1 + \Sigma W_j A_1 F_{1j} = A_1 \epsilon_1 \sigma T_1^4 + (1-\epsilon_1) \Sigma W_j A_1 F_{1j},$$

which must equal $A_1 W_1$ or $\Sigma W_1 A_1 F_{1j}$.

Two results follow:

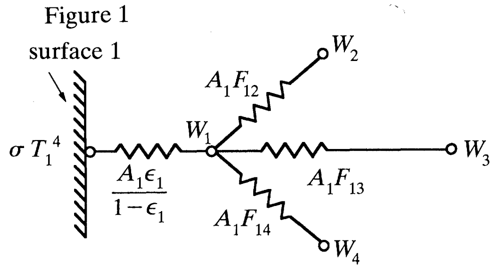
$$Q_1 = A_1 \epsilon_1 (1-\epsilon_1)^{-1} (\sigma T_1^4 - W_1)$$

and

$$Q_1 = \Sigma A_1 F_{1j} (W_1 - W_j).$$

These relations can be put into circuit notation as shown in figure 1.

The driving potentials are the surface emittances, σT_1^4 here or σT_j^4 generally, and the radiosities W_j, which might be thought as having the status of temperatures, not heat flows, despite their units (W m^{-2}). The geometrical conductances associated



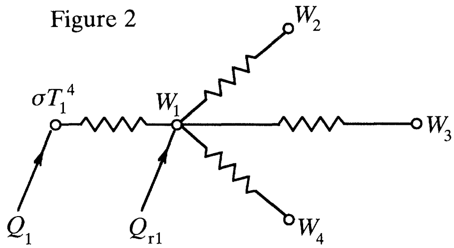


Figure 1. Oppenheim's thermal circuit formulation for radiant exchange at surface 1.

Figure 2. The thermal circuit formulation for radiant exchange at surface 1, together with the radiation Q_{r1} incident upon the surface from an internal radiant source as an input at the radiosity node W_1.

with the surface emissivities are $A_j \epsilon_j/(1-\epsilon_j)$ and the conductances associated with the configuration of the enclosure surfaces one with another are $A_j F_{jk}$.

For a black-body surface, the emissivity is unity, the emissivity conductance is infinite, and the W_j and σT_j^4 nodes coincide.

The Oppenheim formulation is not usually presented so as to include the presence of a source but can be easily modified to do so. Suppose the enclosure contains a source of long-wave radiation which emits Q_r W. The radiant fraction of the heat output from a 100 W bulb is an example. Such a source is driven by a pure heat flow, not by temperature. (Only ambient temperature in fact acts as a pure temperature source.) Thus the surface temperature and area of the source are not relevant to its output of radiation.

Suppose that the flow Q_{r1} falls on surface 1. One approach is to note that the fraction $\alpha = \epsilon$ is absorbed; the fraction $(1-\epsilon)$ is diffusely reflected, is partly absorbed at the other surfaces and is partly reflected so that some fraction is reflected back to surface 1; the process is repeated. The total flow to A_1 is the sum of the absorbed fraction of an infinite number of such multiple reflections.

An alternative approach is derived from the idea that radiosity, the radiant flow from a surface (surface 1 say), in the circuit formulation becomes a flow to the node W_1. The radiant flow Q_{r1} from the source incident upon A_1 is a negatively directed contribution to the radiosity and so the whole of Q_{r1} can be treated as though input at W_1, rather than being part absorbed/part reflected at the surface node σT_1^4 itself.

Thus in this formulation, the net effect Q_j of conduction, convection and surface heating at surface j acts at σT_j^4, and the radiation Q_{rj} from an internal source incident on the surface acts at W_j. See figure 2.

(In Appendix A, the two approaches are shown to lead to the same result.)

(c) *Net conductance between two nodes*

For future development, we need to know the net resistance or conductance between any two nodes j and k describing conditions in a rectangular enclosure.

The view factors between the surfaces of a rectangular room are given below.

The view factor F_{bc} between two adjacent surfaces of dimensions $b \times$ unity and $c \times$ unity is given by

$$F_{bc} = (1/(\pi b))\{\tfrac{1}{4}[b^2 \ln b^2 + c^2 \ln c^2 + (b^2+c^2-1)\ln(b^2+c^2+1) \\ -(b^2-1)\ln(b^2+1) - (c^2-1)\ln(c^2+1) - (b^2+c^2)\ln(b^2+c^2)] \\ + b \arctan(1/b) + c \arctan(1/c) - (b^2+c^2)^{\frac{1}{2}} \arctan(1/(b^2+c^2)^{\frac{1}{2}})\}.$$

The reciprocity relationship $b\,(\text{unity})\,F_{bc} = c\,(\text{unity})\,F_{cb}$ is apparent here.

The view factor F_{bc} between two opposed surfaces in a rectangular room, each of dimensions $b \times c$ and separated by unit distance is

$$F_{bc} = (1/\pi)\{(1/(bc)) \ln [(b^2+1)(c^2+1)/(b^2+c^2+1)] + (2/b)(b^2+1)^{\frac{1}{2}} \arctan [c/(b^2+1)^{\frac{1}{2}}] - (2/b) \arctan (c) + (2/c)(c^2+1)^{\frac{1}{2}} \arctan [b/(c^2+1)^{\frac{1}{2}}] - (2/c) \arctan (b)\}.$$

In this case, $F_{bc} = F_{cb}$. It can be shown that the sum of the view factors from one surface to the other five is unity. There are nine numerically distinct view factors.

The direct geometrical conductance between nodes W_j and W_k is given as

$$G_{jk} = G_{kj} = A_j F_{jk} = A_k F_{kj}.$$

There are 15 such links. Of these,

$$G \text{ (north, floor)} = G \text{ (north, ceiling)} = G \text{ (south, floor)} = G \text{ (south, ceiling)},$$

and there are two further sets of four. On the other hand, G (north, south) is unique. Thus the total of 15 is made up as $3 \times 4 + 3 \times 1$ conductances, and the set contains $3 + 3 = 6$ numerically distinct values.

The overall conductance between nodes j and k, however, is greater than G_{jk} due to the effect of the 14 other paths in series or parallel. To evaluate it, suppose that a heat flow Q_1 due to conduction or surface heating arrives at W_1. Continuity requires that

$$G_{12}(W_1 - W_2) + G_{13}(W_1 - W_3) + G_{14}(W_1 - W_4) + G_{15}(W_1 - W_5) + G_{16}(W_1 - W_6) = Q_1.$$

There are similar relations at the other five W_j nodes so that we have

$$\begin{bmatrix} G_{11} & -G_{12} & -G_{13} & -G_{14} & -G_{15} & -G_{16} \\ -G_{21} & +G_{22} & -G_{23} & -G_{24} & -G_{25} & -G_{26} \\ -G_{31} & -G_{32} & +G_{33} & -G_{34} & -G_{35} & -G_{36} \\ -G_{41} & -G_{42} & -G_{43} & +G_{44} & -G_{45} & -G_{46} \\ -G_{51} & -G_{52} & -G_{53} & -G_{54} & +G_{55} & -G_{56} \\ -G_{61} & -G_{62} & -G_{63} & -G_{64} & -G_{65} & +G_{66} \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \\ W_6 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix},$$

where G_{11} denotes the sum of the conductances attached to node W_1, that is

$$G_{11} = G_{12} + G_{13} + G_{14} + G_{15} + G_{16},$$

and similarly for terms on the principal diagonal. The off-diagonal elements are the negatives of the conductances linking the nodes concerned.

These equations are not linearly independent since if Q_1 to Q_5 were independently assigned, as they can be, Q_6 must by continuity equal $-(Q_1 + Q_2 + Q_3 + Q_4 + Q_5)$. Accordingly we can without loss of generality make $W_6 = 0$, and write

$$\begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} \\ H_{21} & H_{22} & H_{23} & H_{24} & H_{25} \\ H_{31} & H_{32} & H_{33} & H_{34} & H_{35} \\ H_{41} & H_{42} & H_{43} & H_{44} & H_{45} \\ H_{51} & H_{52} & H_{53} & H_{54} & H_{55} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{bmatrix},$$

where the H matrix is the inverse of the G matrix omitting row and column 6.

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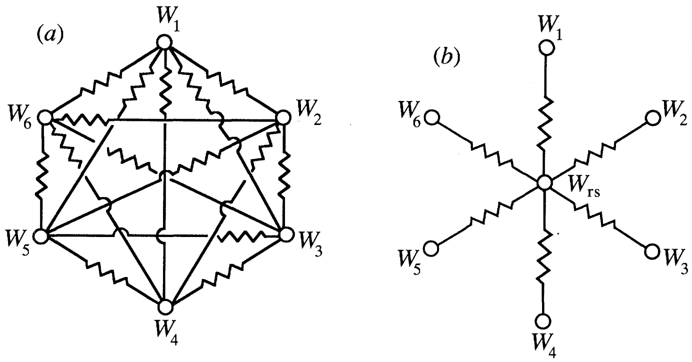

Figure 3. (*a*) Surface to surface radiant links in a black-body rectangular enclosure (the 'delta pattern'). (*b*) Surface to starpoint links.

Now the total resistance between nodes 2 and 4 say is by definition the difference in radiosity between them when $Q_{r2} = +1$ W, $Q_{r4} = -1$ W and $Q_{r1} = Q_{r3} = Q_{r5} = 0$. We denote this resistance by R^{Δ}_{24}. (The Δ denotes the fact that the resistance concerned is that due to a surface to surface pattern of linkages which is a generalized 'delta' configuration.)

Thus $R^{\Delta}_{24} = (W_2 - W_4)/1 = H_{22} - H_{24} - H_{42} + H_{44}$.

Since W_6 is set to zero, the resistance between W_1 and W_6 is found by setting Q_1 to unity and Q_2 to Q_5 to zero.

So $R^{\Delta}_{16} = (W_1 - W_6)/1 = H_{11}$.

The other resistances are found by using one or other of these expressions. There are six numerically distinct values for the resistances. (As an alternative to singling out node 6 for special consideration, we could have supposed instead that node 6 was linked by an arbitrary conductance, external to the radiant network, to a node at zero temperature.)

(*d*) *The star conductances or resistances*

There is an idea of long standing that the exchange of radiation between two surfaces, surfaces 2 and 4 say of a rectangular room, can be estimated by supposing that all radiation is exchanged via a central node, the radiant star node. The exact network and this approximate and quite fictitious equivalent network are shown in figure 3*a*, *b*.

The central question of §2 is: how can the star links be chosen so that the equivalent network best mimicks the external effect of the parent delta network?

Now the flow of energy by radiation from a black-body surface of area A_1 at T_1 to a second black-body surface at T_2 which completely envelops it is

$$Q = A_1\,\sigma(T_1^4 - T_2^4) = A_1(W_1 - W_2),$$

and so the resistance between these nodes is

$$R_1 = (W_1 - W_2)/Q = 1/A_1.$$

It follows that the resistance between W_1 and the radiant star node W_{rs} must be less than this, and we shall write it as β_1/A_1, where of course β_1 is less than 1. The total resistance between nodes 2 and 4 say of a rectangular room is then $\beta_2/A_2 + \beta_4/A_4$, or generally,

$$R^*_{jk} = \beta_j/A_j + \beta_k/A_k.$$

Also $G^*_{jk} = 1/R^*_{jk}$, the star denoting that the resistance or conductance concerned relates to the equivalent star circuit. Again, there are 15 values for R^*_{jk} for a rectangular room.

(e) *The optimal star links*

The β values have to be adjusted so that the external effect of the star circuit resembles that of the delta circuit as closely as possible. This amounts to a generalized delta to star transform. For a 'three surface enclosure' (a very long ridge tent for example), this transform can be exact. It can be shown that

$$\beta_1 = \frac{A_1 G_{23}}{G_{12}G_{23} + G_{23}G_{31} + G_{31}G_{12}}, \quad \text{etc.}$$

If the enclosure is a sphere, so that the areas $A_1, A_2, \ldots$ form patches on its surface, an exact delta to star transform is also possible, regardless of the number of areas into which the spherical surface is divided (see Davies 1990*a*).

For a rectangular enclosure, however, the transform cannot be exact: no star configuration can achieve the same external effect as the parent delta circuit. We require some measure for the difference in their responses and we choose for the purpose the sums of products,

$$S_p = \sum_{j=1}^{6} \sum_{k=j+1}^{6} (G^{\Delta}_{jk} - G^*_{jk})(R^*_{jk} - R^{\Delta}_{jk}),$$

making a total of 15 terms. The expression has the following properties.

1. Either right hand term represents the difference between the exact and star value for the link between nodes j and k.
2. Each product is positive, regardless of whether R^*_{jk} is greater or less than R^{Δ}_{jk}.
3. Each product is dimensionless; it does not favour a difference expressed in either resistance or conductance form.
4. Since S_p represents a sum involving all possible combinations (15) of links between nodes, it provides a single positive measure to express the difference in response between the delta and star configurations.

S_p depends on the values chosen for the βs and by systematically adjusting them, a minimum value for S_p can be found. The resulting star circuit can then be described as the 'optimal' star circuit to express the radiant exchange between the surfaces. There are six β values in a six-sided enclosure but if the enclosure is rectangular, there are only three distinct values. Formally we have to find the values of β_j such that $\partial S_p/\partial \beta_j = 0$, for values of $j = 1, 2, 3$.

The procedure to do this is sketched in Appendix B. It was applied to a series of enclosures of fixed height $h = 1$, of length $l = 10^0, 10^{0.1}, 10^{0.2} \ldots 10^{1.0}$ (a series of 11 values), and depth $d = 10^{-0}, 10^{-0.1}, 10^{-0.2} \ldots 10^{-1.0}$ (also 11 values). When $l = d = h = 1$, the enclosure is of course cubic. With increase of l alone, the enclosure assumes the shape of a square-sectioned corridor; with decrease of d alone, the enclosure becomes square-'tile'-like, and with increase of l and decrease of d, it looks like a plank on edge.

It turns out that β_j values generated by computations on the 121 enclosures are quite closely determined by the fractional area

$$b_j = A_j/(\text{total enclosure area}).$$

The distribution is shown in figure 4.

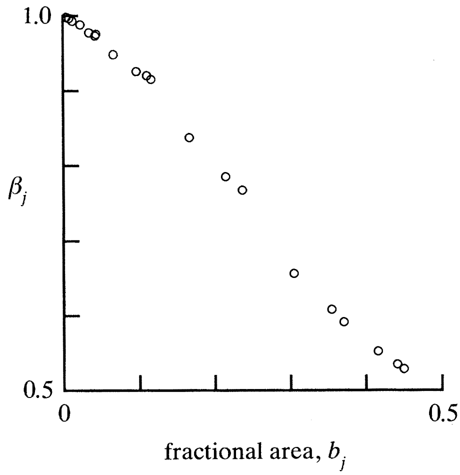

Figure 4. Optimal values of the fractional resistance β_j between the radiosity node W_j and the radiant star node W_{rs} against fractional area $b_j = A_j/\Sigma A_k$. The points refer to enclosures determined by $l/h = 1.000$, 3.162 and 10.000, and $d/h = 1.000$, 0.316 and 0.100, which provide 27 points, some of which are coincident. The full plot of $363\,(=3\times 11^2)$ points shows very little scatter for $0 < b_j < 0.11$ and $0.30 < b_j < 0.45$.

(*f*) *A regression equation for the β values*

A first approximation suggests that $\beta_j \approx 1-b_j$ but it is clear that the β values have some cubic dependence on b_j. To set up a regression equation, we note two limiting conditions.

1. In a room where l and d are each very large in relation to h, the total resistance between floor and ceiling is simply $1/A_{fl}$, so the resistance to the star node is half of this, that is $\beta_{fl} = \frac{1}{2}$. At the same time, A_{fl} tends to half the total area. Thus the regression line must pass through $(b = \frac{1}{2}, \beta = \frac{1}{2})$.

2. If l and d are each very small compared with h, the resistance between the floor and the rest of the enclosure is $1/A_{fl}$ as before, but the resistances between the vertical walls and W_{rs} is much smaller than that between the floor and W_{rs}. Thus the resistance between the floor and W_{rs} itself tends to $1/A_{fl}$. At the same time, the fractional floor area tends to zero. Thus the regression line passes through $(b = 0, \beta = 1)$.

An equation of cubic form satisfying these conditions is of form:

$$\beta_j = 1-b_j+\alpha_1(b_j^2-\tfrac{1}{2}b_j)+\alpha_2(b_j^3-\tfrac{1}{4}b_j).$$

A least-squares fit on the above enclosures provides values for the coefficients α_1 and α_2 and we have

$$\beta_j = 1-b_j-3.54(b_j^2-\tfrac{1}{2}b_j)+5.03(b_j^3-\tfrac{1}{4}b_j).$$

The standard deviation between β_j as found by the optimization procedure (the original values) and β_j values as subsequently estimated from the regression equation is 0.0068.

It should be mentioned that for a cubic enclosure $\beta = \frac{5}{6}$ exactly for each surface. This is easily checked from symmetry. For a cube, this value should be used and the cubic case was excluded in evaluating the regression coefficients. In fact the coefficients are practically the same with or without the cubic case, but the regression

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equation provides a relatively bad estimate for β for a cube, 0.844 as against the correct value of 0.833.

(g) *How good is the transformation?*

Although the above procedure yields the 'best' star configuration, that does not of itself mean that the star circuit closely mimicks the delta form. Two checks are available; in each case we can use either the β_j (original) values or the β_j (estimated) values.

(i) *The 'two temperature' enclosure*

Consider an enclosure with black-body surfaces. Suppose that one surface, the floor say, is at one temperature and all the others are at some other common temperature. In this case, the conductance between the floor and the common surface is A_{fl}.

We can write down an estimate of this quantity as found by using β values. We define G_{rs} as the total conductance from W_{rs} to the enclosure:

$$G_{\mathrm{rs}} = 2(A_1/\beta_1 + A_2/\beta_2 + A_3/\beta_3).$$

The difference between the estimated and exact conductances between the floor and the common surface is then

$$[(A_1/\beta_1)^{-1} + (G_{\mathrm{rs}} - A_1/\beta_1)^{-1}]^{-1} - A_1$$

and the difference in resistances is

$$[(A_1/\beta_1)^{-1} + (G_{\mathrm{rs}} - A_1/\beta_1)^{-1}] - 1/A_1.$$

The negative of the product of these quantities,

$$p_1 = \frac{\beta_1 G_{\mathrm{rs}}}{G_{\mathrm{rs}} - A_1/\beta_1} + \frac{G_{\mathrm{rs}} - A_1/\beta_1}{\beta_1 G_{\mathrm{rs}}} - 2,$$

provides a non-dimensional, necessarily positive, measure of the difference. There are three such quantities for the enclosure, associated with the floor, the north and the east walls (the ceiling difference is identical with the floor difference and similarly for the south and west walls). The quantity

$$\delta_2 = (\tfrac{1}{3}(p_1 + p_2 + p_3))^{\frac{1}{2}}$$

is the root mean product deviation between the exact and star versions of the link between one surface and the remaining surfaces of the enclosure. The subscript 2 corresponds to the 'two-temperature' enclosure. δ should have a further subscript to denote whether it is based on the original β values (subscript o), or on estimated β values (subscript e).

(ii) *The 'six-temperature enclosure'*

In the above test, five of the six star links were lumped to find the link between W_{rs} and the common surface. A more severe test is that based on the difference between the 15 individual pairs of nodes as expressed by S_{p} above. We can define a further δ as

$$\delta_6 = (\tfrac{1}{15} S_{\mathrm{p}})^{\frac{1}{2}},$$

which is of course the root mean product deviation between the exact and star versions of the net link between node j and node k. The subscript 6 denotes that links are considered between all 6 nodes individually, and again it can be based on original and estimated β values.

Table 1. *Various measures of the accuracy with which a star circuit can represent radiant exchange in a rectangular room*

(The measures in each column are δ_{2o}, δ_{2e} (for a 'two-temperature' enclosure), δ_{6o}, δ_{6e} (for a 'six-temperature' enclosure) and Δ_{max} and Δ_{min} for the six-temperature enclosure. Values are in percent.)

l/h:	1.00	1.58	2.51	3.98	6.31	10.00
d/h						
1.00	0.00	0.07	0.20	0.30	0.37	0.43
	0.00	1.10	0.76	0.51	0.64	0.90
	0.01	0.97	1.64	2.08	2.37	2.56
	0.01	1.42	1.75	2.13	2.48	2.76
	0.00	2.50	3.44	3.93	4.12	4.16
	−0.01	−0.94	−1.90	−2.64	−3.62	−3.33
0.63	0.03	0.09	0.19	0.27	0.30	0.30
	1.12	0.90	0.65	0.65	0.85	1.05
	1.05	1.53	1.86	2.08	2.24	2.35
	1.49	1.72	1.95	2.20	2.42	2.61
	3.52	4.08	4.13	3.92	3.79	4.04
	−0.00	−1.42	−2.00	−2.96	−3.68	−4.18
0.40	0.10	0.19	0.25	0.26	0.23	0.18
	0.84	0.65	0.59	0.70	0.87	1.01
	1.72	1.83	1.80	1.76	1.77	1.81
	1.87	1.92	1.91	1.92	1.97	2.04
	4.15	3.73	3.13	2.74	3.25	3.57
	−0.59	−1.42	−2.11	−2.63	−3.01	−3.26
0.25	0.29	0.35	0.35	0.31	0.26	0.23
	0.45	0.31	0.41	0.58	0.72	0.81
	1.86	1.74	1.53	1.36	1.28	1.27
	1.92	1.79	1.61	1.47	1.41	1.40
	3.15	2.91	2.34	2.20	2.66	2.95
	−0.99	−1.38	−1.67	−1.87	−2.00	−2.10
0.16	0.41	0.42	0.38	0.32	0.27	0.24
	0.08	0.31	0.49	0.61	0.68	0.73
	1.66	1.46	1.21	1.01	0.90	0.86
	1.70	1.52	1.29	1.10	0.98	0.94
	3.04	2.70	2.14	1.71	2.06	2.29
	−0.88	−0.99	−1.06	−1.10	−1.13	−1.15
0.10	0.44	0.42	0.36	0.29	0.24	0.20
	0.45	0.56	0.62	0.66	0.67	0.68
	1.36	1.16	0.92	0.73	0.62	0.57
	1.43	1.25	1.03	0.84	0.72	0.66
	2.67	2.31	1.80	1.30	1.52	1.67
	−0.58	−0.58	−0.77	−0.89	−0.95	−0.99

It is also useful to know what the largest fractional error in the links may be. We define Δ as $(R^{\Delta}_{jk}-R^{*}_{jk})/R^{\Delta}_{jk}$. Δ is numerically small and may be positive or negative and we define the extreme values as Δ_{max} and Δ_{min}.

Values for all these measures – δ_{2o}, δ_{2e}, δ_{6o}, δ_{6e}, Δ_{max} and Δ_{min} – are given in table 1 for half the set of l and half the set of d values mentioned above. All values are expressed in percent. (Thus S_p for a cube is found to be 8.0×10^{-8} and the value of δ_{6o} as a percentage and expressed to two decimals becomes 0.01 as shown in table 1.)

(h) Discussion

As will be seen from the first two entries for each enclosure in table 1, the star circuit represents the behaviour of the parent well when we are concerned only with the exchange between one surface and the other five taken to be at a common temperature. Errors amount to little more than 1%. As would be expected, the error is usually a little larger with estimated than with original β values, but there is no logical reason why this must be so.

As regards the links between individual nodes (entries 3 and 4), average fractional errors of up to 3% are found. δ_6 values are greater than δ_2 values since positive and negative differences tend to cancel in forming δ_2, but not in forming δ_6. Clearly, a value of δ_6 found by using estimated β values must now be greater than δ_6 using the original optimal β values and this will be seen to be the case, though the deterioration is not very marked.

The extreme fractional errors (entries 5 and 6) must be greater than what is virtually their root mean square value, δ_{6e}, and again the table bears this out.

The fractional errors shown in the table are quite modest in relation to the uncertainties that appear in analyses of building heat transfer and the delta-to-star transformation can be described as satisfactory. I did not develop this method, however, to simplify radiant exchange as such: the exact approach does not present any particular difficulties and a simplification is not needed. Rather, this procedure is the first step toward the evolution of a room global temperature and before this can be done, we have to examine what meaning can be attached to the radiant star node W_{rs} which so far cannot be said to have any physical status. The star circuit cannot provide any facilities not provided by the parent circuit; thus W_{rs} is not accessible and long-wave radiation, which can legally be treated as input at the W_j nodes, cannot be input at W_{rs}. That is in effect, however, what we are going to do, and its justification is the task for §3. Before doing that, however, it is convenient to linearize the driving potentials.

(i) Linearization of the driving potentials

According to conventional theory, heat flow through solids is strictly proportional to temperature difference, and convective transfer too is at any rate approximately proportional to this difference. Long-wave radiant transfer, however, is proportional to differences in (temperature)^4. In order that mixed mode transfer calculations can be made without complication in room heat transfer, it is usual to suppose that radiant exchange too is proportional to difference in (temperature)^1, and provided these differences are not large, the practice is satisfactory. (Any error is simply one more in calculations fraught with uncertainty.) Two steps are needed.

1. From each driving potential σT_j^4 or W_j, subtract some representative room potential, W_{rs}, say, if we are handling a star circuit. This leaves the heat flows in all links unaltered.

2. We can express W_j as σT_{jb}^4 where T_{jb} is the black-body equivalent temperature of surface j, and W_{rs} as σT_{rs}^4. A heat flow Q_j can thus be expressed as

$$\begin{aligned} Q_j &= A_j(W_j - W_{rs}) = A_j\,\sigma(T_{jb}^4 - T_{rs}^4) \\ &= A_j[\sigma(T_{jb}+T_{rs})(T_{jb}^2+T_{rs}^2)](T_{jb}-T_{rs}) \\ &= A_j\,h_{rj}(T_{jb}-T_{rs}), \end{aligned}$$

where $$h_{rj} = [\sigma(T_{jb}+T_{rs})(T_{jb}^2+T_{rs}^2)].$$

No approximation has been made, and the heat flow is now proportional to a temperature difference, $(T_{jb}-T_{rs})$. However, h_{rj} depends on T_{rs}, a global variable, and upon the special temperature T_{jb}. Some T_{jb} values must lie above and some below T_{rs}, but this variation is small compared with their absolute values and if this is ignored, we can assume a global value of h_r as

$$h_r = 4\sigma T_{rs}^3.$$

With a typical room value of T_{rs} of 20 °C or 293 K, h_r is about 5.7 W m^{-2} K^{-1}.

Division of the driving potential difference by $4\sigma T_{rs}^3$ implies that we must multiply the conductances, hitherto expressed in units of m^2, by $\sigma 4T_{rs}^3$ or h_r, so as to give the conductances in the physical units W K^{-1}. This applies of course to both the conductances deriving from room shape (the product of the delta to star transform) and the emissivity conductances, which are the same in either formulation.

It will be convenient from now on to work in terms of linear temperatures and physical conductances. The total physical conductance between the surface node T_j and T_{rs} will be denoted by S_j (units W K^{-1}).

$$\frac{1}{S_j} = \frac{1-\epsilon_j}{A_j\,\epsilon_j\,h_r} + \frac{\beta_j}{A_j\,h_r},$$

or $$S_j = A_j E_j h_r, \quad \text{where} \quad 1/E_j = (1-\epsilon_j)/\epsilon_j + \beta_j.$$

3. The space-averaged observable radiant temperature

The previous section dealt with a transformation of the radiant exchange in an enclosure, leading to the formulation of a radiant star temperature, T_{rs}. As was mentioned, the quantity W_{rs} and the temperature T_{rs} derived from it, are fictitious constructs and have no physical significance. Yet T_{rs} clearly must be some function of the radiant field. This section is concerned with the question as to what physical significance, if any, we may attach to it. It will be demonstrated that T_{rs} may serve as an estimate of the average observable radiant temperature T_{rv} in the enclosure.

First, a note about 'large' and 'small' conductances. The floor of a room might have an area of say 20 m^2. With a value of h_r of 5.7 W m^{-2} K^{-1} and an emissivity of 0.9, the value of S_{fl} would be around 100 W K^{-1}. The floor convective conductance is likely to be around 60 W K^{-1}. These conductances will be regarded as 'large'. Now temperature measurement is typically made with small devices. The radiant conductance between a thermometer bulb of 2 mm diameter and the room is of order 0.0001 W K^{-1} and the convective conductance is of the same order. These will be regarded as 'small'. The areas associated with room furnishings and with occupants too can be regarded as 'small' compared with surface areas of the room itself. Perceptions of thermal comfort, whether reported by subjects or related to objectively observed measures, must thus be associated with links which are very much smaller than those concerned with the bulk movement of heat, of order kilowatts, within and from the room. Although studies in human thermal comfort have been conducted since the 1920s, attention does not appear to have been drawn to this distinction. For consider 'dry resultant temperature' T_c, which is taken as a measure of the comfort temperature in a room. It is usually expressed as

$$T_c = \tfrac{1}{2}T_a + \tfrac{1}{2}T_r,$$

where T_a is the mean air and T_r the mean radiant temperature in the room, without any further speculation of just how T_c is linked to these quantities. If the expression is put into thermal circuit form, T_c could be modelled as linked to T_a and T_r either by two equal and large conductances, or by two equal and small conductances. We can reject the large conductance case, however, by noting two points.

1. The mean radiant and air temperatures in a room only interact via their respective exchanges with the large bounding areas of the room; their interaction via furnishings, as distinct from bounding surfaces, is much smaller; large conductances to T_c in a thermal circuit would provide an invalid additional link between T_a and T_r.

2. A large heat input, several kilowatts, may be expected to lead to a modest rise in either air or surface temperatures in a room; by contrast, the application of 1 kW to a thermometer bulb will generate a very high temperature there. Clearly, if T_c is to be included in a thermal circuit, it must be linked by low conductances consistent with generation of a high temperature if such a heat flow were applied at T_c.

Thus we have to describe T_c as a low conductance or high impedance node, while the nodes describing surface and air temperatures are high conductance, low impedance nodes.

(a) *The average observable radiant temperature in an enclosure due to an internal radiant source*

To arrive at a measure of the radiant temperature in a room, we suppose that the walls are black-body surfaces at a reference temperature of zero. The room will be supposed to be air-free to remove the question of convective exchange. A sensor such as a thermometer placed anywhere in the room will register a local or point radiant temperature T_{rp} of zero. Suppose now that a source of longwave radiation of strength Q_r W is placed say at the centre of the room. The sensor will now record a high temperature when close to the source and a low value when remote from it; there must be a space or volume-averaged value T_{rv}. To find T_{rv}, we suppose the sensor to be spherical of radius r so that it intercepts radiation from the source on a cross section πr^2 and absorbs a fraction α. It reradiates this energy to the walls at zero from its full area of $4\pi r^2$. Now the intensity of radiation passing through a spherical surface of radius R centred on the source is $Q_r/(4\pi R^2)$. The temperature T_{rp} recorded at a point distance R from the source is given by the relation

$$(Q_r/4\pi R^2)\,\pi r^2\alpha = \epsilon 4\pi r^2 h_r(T_{rp}-0).$$

The volume-averaged value of T_{rp}, T_{rv}, is found by summing such values over locations in a regular three-dimensional grid, or more generally as

$$T_{rv} = \iiint T_{rp}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \Big/ \iiint \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z.$$

It is convenient to remove the effect of the size of the enclosure by expressing T_{rv} in non-dimensional form as

$$\beta_{rv} = \frac{T_{rv}\,h_r}{Q_r}\Sigma A,$$

where ΣA denotes the total area of the six surfaces of the enclosure. Since $R^2 = x^2+y^2+z^2$, measured from the source, and $\alpha = \epsilon$, we have

$$\beta_{rv} = \Sigma A \iiint \frac{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z}{(x^2+y^2+z^2)} \Big/ 16\pi \iiint \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z.$$

Table 2. *Values for the non-dimensionalized observable radiant temperature due to a radiant source* Q_r *in an enclosure*

(First row, β_{rv} (centre), source at centre; second row, β_{rv} (S wall), source at the centre of the $l \times h$ wall; third row, β_{rv} (W wall), source at the centre of the $d \times h$ wall; fourth row, β_{rs}, the value found by supposing that Q_r is input at T_{rs}. For enclosures above the line, β_{rv} (S wall) < β_{rs} < β_{rv} (centre).)

l/h:	1.00	1.58	2.51	3.98	6.30	10.000
d/h						
1.00	0.916	0.915	0.910	0.903	0.897	0.892
	0.548	0.566	0.580	0.590	0.596	0.598
	0.548	0.515	0.491	0.474	0.462	0.454
	0.843	0.832	0.810	0.789	0.771	0.759
0.630	0.916	0.915	0.911	0.906	0.902	0.898
	0.588	0.607	0.621	0.629	0.633	0.635
	0.531	0.504	0.484	0.471	0.462	0.456
	0.827	0.805	0.780	0.758	0.743	0.732
0.398	0.921	0.925	0.925	0.924	0.923	0.922
	0.635	0.656	0.670	0.678	0.682	0.685
	0.520	0.501	0.487	0.477	0.471	0.466
	0.777	0.746	0.719	0.699	0.686	0.677
0.251	0.939	0.950	0.956	0.960	0.962	0.962
	0.688	0.712	0.727	0.737	0.743	0.746
	0.520	0.508	0.499	0.493	0.489	0.486
	0.708	0.676	0.652	0.636	0.626	0.620
0.158	0.973	0.992	1.004	1.012	1.017	1.020
	0.748	0.776	0.795	0.806	0.814	0.818
	0.531	0.525	0.521	0.518	0.516	0.515
	0.640	0.615	0.598	0.587	0.580	0.576
0.100	1.024	1.050	1.068	1.080	1.087	1.091
	0.818	0.849	0.871	0.885	0.894	0.900
	0.553	0.552	0.551	0.551	0.550	0.550
	0.589	0.572	0.561	0.554	0.549	0.547

Integration takes place over the volume of the enclosure. If the source is placed in the centre, the limits are $-\frac{1}{2}l$ to $+\frac{1}{2}l$, $-\frac{1}{2}d$ to $+\frac{1}{2}d$ and $-\frac{1}{2}h$ to $+\frac{1}{2}h$. If the source is placed in the middle of say the $d \times h$ wall, the limits for l become 0 to 1. β_{rv} is a purely geometrical quantity.

Message has recently derived an analytical expression for β_{rv} (Davies & Message 1992). Values for various shaped enclosures with the source Q_r at the centre are given as the first entry in table 2.

As the source is moved progressively towards one wall, an increasingly large fraction of the enclosure is located further from the source so the average temperature falls. The β_{rv} values for a wall-mounted source are given as the second and third entries in the table. Intermediate values for a cubic enclosure are given as the first entry in table 3.

A panel radiator is a distributed rather than a point source. To see what effect the finite extent of a source might have, the calculations were repeated replacing the single source Q_r by an array of nine sources forming a square in a vertical plane. Each was of strength $\frac{1}{9}Q_r$, one was at the centre of the square, one at each of its corners and one at the centre of each side. The square was of area $(\frac{1}{5}d)(\frac{1}{5}h)$, its centre was at a

Table 3. *Variation of β_{rv} as the position of the radiant source is moved from the left wall to the right wall of a cubic enclosure*

fractional position	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
β_{rv} (point source)	0.547	0.740	0.829	0.880	0.907	0.915	0.907	0.880	0.829	0.740	0.547
β_{rv} (9 point sources)	0.544	0.735	0.824	0.874	0.901	0.909	0.901	0.874	0.824	0.735	0.544

height of $\frac{1}{2}h$ and distant $\frac{1}{2}d$ from the $l \times h$ wall and taken to be located at a series of distances from the $d \times h$ wall. The 9-point β_{rv} values too are shown in the table. They differ little from the single point source values and it is clear that average radiant temperature in the room depends relatively little on the shape of the source.

(*b*) *The radiant star temperature due to an internal radiant source*

As discussed earlier, the radiant heat from an internal source is to be treated as though input at the T_j nodes for a black-body enclosure, or at the T_{jb} nodes if the surfaces are grey. It is meaningless to suppose that it is input at T_{rs}. If, however, we were to treat Q_r as though input at T_{rs}, the temperature it would generate there in an enclosure with black-body surfaces at zero reference temperature is simply $T_{rs} = Q_r/(\Sigma A_j h_r/\beta_j)$. The non-dimensional value of this quantity can be written as

$$\beta_{rs} = \Sigma A_j/(\Sigma A_j/\beta_j),$$

which is again a purely geometrical quantity. Its values are given as the fourth entry in table 2. It will be seen that β_{rs} lies between the values of β_{rv} (source at centre) and β_{rv} (source at wall) for the higher values of d/h.

(*c*) *The radiant temperatures due to bounding surfaces*

It is clear that a sensor at some specified position, near the floor say, in an enclosure can be modelled as shown in figure 5.

All the radiant conductances are small in the sense outlined above, but they will be proportional to the solid angles the sensor subtends to the respective surfaces. Thus the conductance to the floor might be relatively large. If the surfaces are at different temperatures, T_{rp} will vary from point to point. One might guess that the average value, T_{rv}, might be estimated by the value of T_{rs} which is linked to the walls by the large conductances of figure 3*b*.

This point has been examined recently by Davies & Message from which table 4 is reproduced. It includes a range of rectangular shapes: with $h = 1$ in all cases,

$l = 1$ and $d = 1$ describes a cube,

$l = 0.1$ and $d = 0.1$, a telephone kiosk,

$l = 1$ and $d = 0.1$, a vertical square tile-like enclosure,

$l = 10$ and $d = 0.1$, an enclosure like a plank on edge,

$l = 10$ and $d = 1$, a long square-sectioned corridor, and

$l = 10$ and $d = 10$, a horizontal square tile-like enclosure.

When $d/h = 1$, both γs increase moderately with increasing l/h; this must be so since the floor area increases as a fraction of the total area. When d/h decreases, the γs similarly decrease.

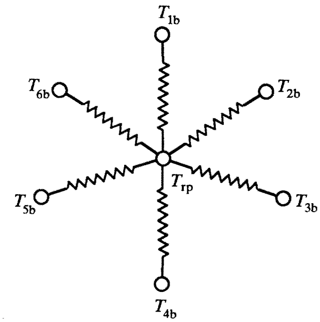

Figure 5. The radiant thermal circuit for a sensor in a room. (Compare this with figure 3*b*.)

Table 4. *Values of temperature within a black-body rectangular enclosure in which the floor is maintained at unit temperature T and the remaining surfaces are at zero*

(The upper value is the non-dimensionalized volume averaged observable radiant temperature, $\gamma_{rv} = T_{rv}/T$, and the lower the non-dimensionalized radiant star temperature, $\gamma_{rs} = T_{rs}/T$.)

l/h:	0.100	0.158	0.251	0.398	0.631	1.00	1.58	2.51	3.98	6.30	10.000
d/h											
10.00											0.403
											0.450
6.31										0.368	0.384
										0.420	0.434
3.98									0.325	0.345	0.359
									0.374	0.396	0.410
2.51								0.275	0.298	0.314	0.325
								0.310	0.340	0.359	0.371
1.58							0.220	0.245	0.263	0.276	0.284
							0.237	0.268	0.290	0.304	0.314
1.00						0.167	0.190	0.209	0.223	0.232	0.239
						0.167	0.194	0.214	0.227	0.235	0.241
0.630					0.119	0.140	0.157	0.171	0.181	0.187	0.192
					0.111	0.132	0.146	0.155	0.161	0.165	0.167
0.398				0.082	0.098	0.113	0.125	0.134	0.141	0.146	0.149
				0.072	0.086	0.095	0.100	0.103	0.105	0.106	0.107
0.251			0.054	0.066	0.077	0.088	0.096	0.102	0.107	0.110	0.112
			0.046	0.055	0.060	0.063	0.065	0.065	0.066	0.066	0.066
0.158		0.035	0.043	0.052	0.059	0.066	0.072	0.076	0.079	0.081	0.082
		0.029	0.035	0.038	0.039	0.040	0.041	0.041	0.041	0.041	0.041
0.100	0.023	0.028	0.034	0.039	0.044	0.049	0.052	0.055	0.057	0.058	0.059
	0.018	0.022	0.024	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025

The table demonstrates broad agreement between γ_{rv} and γ_{rs}, that is, T_{rv} and T_{rs}, for those enclosures where d/h is not much less than 1. Some degree of agreement might be regarded as intuitively obvious, but intuition is not a satisfactory guide. For consider an enclosure having the shape of a very long and squat ridge tent, so that it has effectively three isothermal surfaces exchanging radiation: the base, and the two sloping sides whose combined area is little greater than that of the base. If the base is at unit temperature and the sides at zero, the average observable radiant temperature T_{rv} must be somewhere near 0.5. Now for such an enclosure, a delta to star transform is possible exactly, and when this is done, it turns out that for quite straightforward reasons, the radiant star temperature T_{rs} tends to that of the base, unity. Thus in this case T_{rs} is approaching double the value of T_{rv} and so we cannot assume automatically that T_{rs} must be a satisfactory approximation to T_{rv}: a sufficient degree of agreement has to be demonstrated. The table indicates the degree of agreement found.

(d) *A star-based model for radiant exchange in a room*

To have a simple model for radiant exchange for plant design purposes, it would appear from these results that β_{rs} may serve as an estimate of β_{rv}, that is to say, we can estimate the average radiant temperature in an enclosure from the radiant star temperature, assuming that the radiant input Q_r from an internal source acts at T_{rs}. The emissivities of the surfaces can readily be taken into account by inclusion of their conductances in series with the star links.

T_{rs} will not be a very accurate estimate for T_{rv} as the values in tables 2 and 4 show, and some further caveats have to be noted.

1. Of the radiant input to an enclosure, only that part that traverses the occupiable space and can be intercepted by sensors or occupants should be treated as input at T_{rs}. The radiation from the back of a wall-mounted radiator should be treated as though input at the T_{jb} node for that wall and not at T_{rs}.

2. The local observable radiant temperature T_{rp} is not itself a unique quantity but depends on the shape of the sensor: if the sensor is flat (not spherical as assumed above) and the flat surface faces the source, it will record a higher temperature than that when it is placed edge-on to the source. If the two surfaces of such a sensor were not black-body but had different emissivities, we should have a further variation of observed temperature with the orientation of the sensor. Thus T_{rv} depends upon both the position of the source and the characteristics of the sensor.

Clearly, radiant studies can be conducted to take account of a great part of the fine detail in a room, but it may not be necessary for the design of heating and cooling appliances in most applications. A model somewhat along the lines of the above model has been tacitly assumed for this purpose in the past and the present argument serves to clarify its details.

(e) *The star-based model for convective exchange in a room*

Air movement in a room takes place under the action of infiltration forces (wind effects and mechanical ventilation), under the action of the very weak buoyancy forces arising from small temperature differences in the room (occupants, floor, ceiling, etc.), and under the action of the rather stronger buoyancy forces on air warmed at intentionally heated surfaces such as radiators. The movement is complicated and has been the subject of a number of computational fluid dynamical investigations in recent years.

For simple design purposes, however, it is usual to work in terms of a volume-averaged air temperature T_{av} (related to local or point values T_{ap} as T_{rv} is to T_{rp}), and to assume appropriate convective heat transfer coefficients h_{cj} between T_{av} and the various surface nodes at T_j. Working values of h_{cj} are often taken to vary between about $\frac{3}{2}$ and $\frac{9}{2}$ W m^{-2} K^{-1}. The part of the heat input to the room that is input into the air, Q_a, is taken to act as a whole at the air temperature T_{av}. This model is also a star-based model.

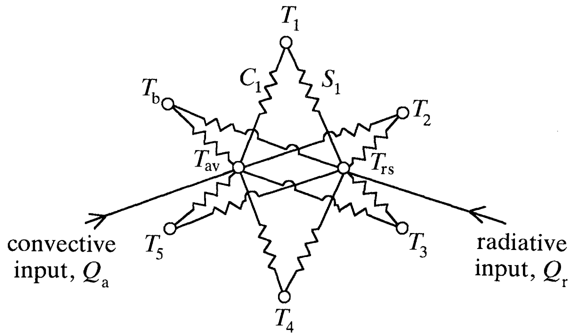

Figure 6. The internal heat exchanges C_j and S_j for the two-star model which centre on the air and radiant temperatures T_{av} and T_{rs}.

(*f*) *The two-star model for total exchange in a room*

The total exchange in a room consists of the long-wave radiant and convective exchanges, together with shortwave input, mainly solar. A model for the first two of these simply consists in the superposition of the star-based radiant model and the convective model so as to form the two-star model shown in figure 6.

A radiant input Q_r acts at T_{rs}, and a convective input Q_a at T_{av}. Dry resultant temperature T_c with its low conductance links to T_{rs} and T_{av} can be included. The model provides an acceptably detailed model for design purposes. To it of course must be added the conductances associated with wall conduction and storage, and the links associated with air movement. The ingress of air should strictly be modelled as a 'quasi-heat source' of strength $vs\,(T_{sup}-T_{av})$ W, where v denotes the ingress volume flow rate ($m^3\ s^{-1}$), s is the volumetric specific heat of air usually taken as 1200 J $m^{-3}\ K^{-1}$, and T_{sup} the temperature of the incoming air, whether as a heated or cooled ducted supply, by infiltration from an adjacent room, or by infiltration from ambient. If the air infiltrates from ambient, however, the link can simply be modelled as the conductance V W K^{-1} and will be modelled in this way later.

(*g*) *Handling solar gains*

Much of the solar radiation entering a room through a window falls on the floor and the remainder falls on walls and furnishings. It is partly absorbed at these surfaces and partly reflected. Specular reflections apart, this process can be modelled as was explained for the long-wave gains: the radiation falling on the floor ($Q_{sol,\,fl}$) is to be totally absorbed at a W_{fl}-like node. W_{fl} is linked to the floor by an absorptivity conductance based on its short-wave absorptivity (replacing the earlier long-wave emissivity or absorptivity) and to the other W_j nodes by A_j/β_j conductances. The emittances of all the room surfaces for short-wave emission are zero so simple network analysis provides values for the W_js. Thus the total solar radiation physically falling on the floor generates a radiosity-like quantity W_{fl}. The actual short-wave heat gain Q_{sw} at the floor is $W_{fl}\,A_{fl}\,\alpha_{fl}/(1-\alpha_{fl})$. This Q_{sw} then acts as a secondary pure heat source at T_{fl}, and is lost by the usual processes of conduction/storage, convection and long-wave radiation. The solar radiation diffusely reflected from the floor can be handled by supposing that W_{fl} is linked to W_{ceil}, etc., by a delta or star-type network.

4. Merging the radiation and convection mechanisms

The contentions of the last section led us to the position where we could imagine room heat transfer taking place under the action of the quite independent processes of radiation and convection; they are 'driven' by separate temperatures T_{rs} (as proxy for T_{rv}) and T_{av}, through conductances S_j and C_j, and it is only at solid surfaces that they interact. Thus the heat flow from the room as a whole to the inner surface of some wall (subsequently to be lost by conduction) is given as

$$Q_{cond} = S_{wall}(T_{rs} - T_{wall}) + C_{wall}(T_{av} - T_{wall}).$$

Up to the 1960s, design methods for room heat transfer were only concerned with two processes: the loss of heat from a room by ventilation and the steady-state loss by conduction through an outer wall. The conduction loss through a wall of area A_{wall} was computed from an expression of type

$$Q_{cond} = A_{wall}\, u_{wall}(T'_a - T_o).$$

Here T'_a is the general measure of the room temperature referred to in §1, T_o is ambient temperature and u_{wall} is the transmittance (W m^{-2} K^{-1}) of the wall, given as

$$\frac{1}{u_{wall}} = \frac{1}{h'_r + h_c} + \frac{d}{\lambda} + \frac{1}{h_{co}},$$

where h'_r is the radiation coefficient (possibly including mention of surface emissivity), h_c the inside convection coefficient, d and λ the thickness and conductivity of the wall material (this term must be summed for a multilayer wall), and h_{co} the outside surface coefficient, typically of order 20 W m^{-2} K^{-1}.

The wall construction and outer coefficient are not of current interest. It will be seen, however, that the heat flow from the room as a whole to the inner surface of the outer wall in question (at T_{wall}) has been treated as though driven by the single index temperature T'_a through the merged radiative/convective conductance, $A_{wall}(h'_r + h_c)$:

$$Q_{cond} = A_{wall}(h'_r + h_c)(T'_a - T_{wall}).$$

This form was justified in a loose way by saying that the room air and internal surfaces (that is, all room surfaces apart from the one under consideration) were at a common temperature, T'_a, which drives both radiation and convection. All heat input, both radiative and convective, was to be taken to be input at T'_a. This is not a logically sustainable argument but the computational procedure based on it works well enough. The question thus poses itself: can we derive a form of index temperature based on both radiant and air temperatures which serves to drive a merged radiant and convective energy flow to a bounding surface? Can we find logically acceptable forms for T'_a and for $(h'_r + h_c)$?

It is the aim of this section to demonstrate that this is possible, though not in general exactly, and to examine the properties of the index, the 'rad-air' temperature T_{ra} so arrived at.

(a) *The equivalence theorem*

The mergence depends upon a circuit equivalence theorem. Consider the simple circuit of figure 7*a*, consisting of air temperature T_{av} held at some value by a pure temperature source, linked to a surface at T_1 via a convective link C_1 and further to

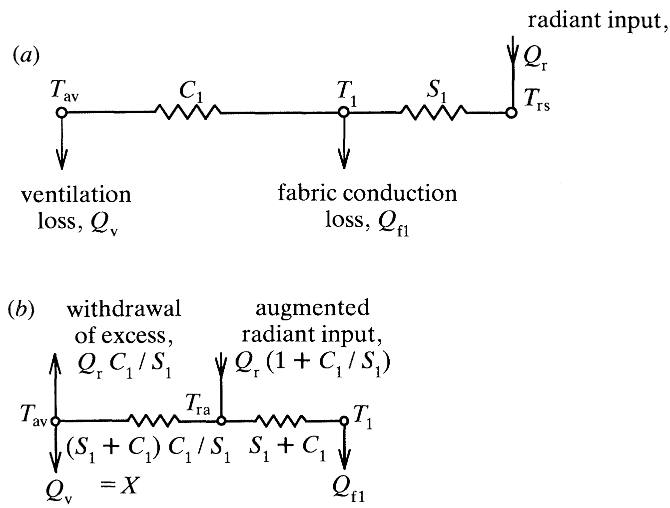

Figure 7. The two circuits to illustrate the equivalence theorem.

the radiant star temperature T_{rs} through the radiant link S_1. The surface 1 is taken to envelop an entire enclosure, so that if the enclosure is rectangular in shape, all six surfaces have the common temperature T_1. A radiant heat source Q_r acts at T_{rs}. Some of the input is lost by fabric conduction Q_{f1} from T_1 and the remainder, $Q_v = Q_r - Q_{f1}$, from T_{av} by ventilation. This is the two-star model for a single surface enclosure; it is not necessary at this stage to include the fabric conduction conductance F_1 which describes the heat loss from T_1, nor the ventilation conductance V describing the ventilation.

It is clear that
$$T_1 = T_{av} + (Q_r - Q_{f1})/C_1$$
and that
$$T_{rs} = T_{av} + (Q_r - Q_{f1})/C_1 + Q_r/S_1.$$

Now consider the circuit of figure 7*b*, which is similar to that of figure 7*a* except that T_{rs} and S_1 and Q_r are not present. Instead a new node T_{ra} is located on C_1 defining segment conductances X and Y such that $X/Y = C_1/S_1\,(=\alpha_1$ say); thus the information on S_1 is not lost.

Since $1/X + 1/Y = 1/C_1$, it follows that
$$X = (S_1 + C_1)\,C_1/S_1\,(=(1+\alpha_1)\,C_1)$$
and
$$Y = S_1 + C_1\,(=(1+\alpha_1)\,S_1).$$

The heat input is handled as an augmented input $Q_r(1+\alpha_1)$ at T_{ra} and a withdrawal of the excess, $Q_r\alpha_1$, from T_{av}; thus the total input remains the same.

The losses from T_{av} and T_1 remain as before. Then
$$T_{ra} = T_{av} + [Q_r(1+\alpha_1) - Q_{f1}]/X.$$

The temperature T_1' at the T_1 node, possibly different from its above value, is given as
$$T_1' = T_{av} + [Q_r(1+\alpha_1) - Q_{f1}]/X - Q_{f1}/Y,$$

which in fact will be found to equal T_1.

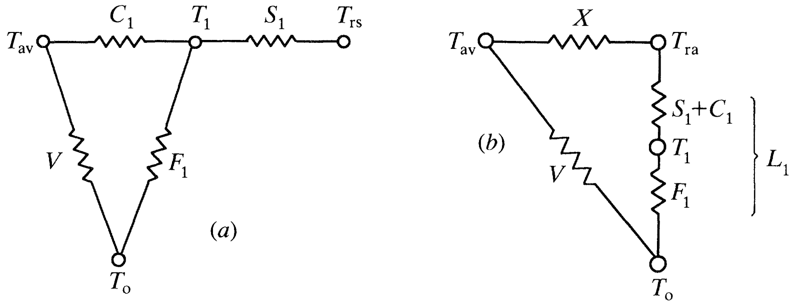

Figure 8. (a) The basic thermal circuit for an enclosure. (b) The equivalent thermal circuit.

Further, it will be found that T_{rs}, not explicitly included in the X, Y formulation, can be found from it as

$$T_{rs} = T_{ra}(1+\alpha_1) - T_{av}\,\alpha_1.$$

The C, S formulation (figure 7a) provides values of temperature at the radiant star and surface nodes due to a heat input at T_{rs}. The X, Y formulation (figure 7b) provides exactly the same information. Thus the one is the transform of the other.

Before considering the significance of this result it is of interest to see it in relation to Norton's theorem. Norton's theorem states, replacing electrical by thermal terminology, that a circuit of arbitrary complexity connected between two nodes can be replaced by a pure heat source Q_N in parallel with a conductance C_N connected between the two nodes.

In the present case, the two nodes concerned are T_{av} and T_1, which differ in temperature due to a heat input Q_r. Both the C, S and the X, Y formulations provide valid, if trivial, examples of Norton's theorem. The present equivalence theorem however addresses a different issue. Norton's theorem serves to reduce a circuit of any degree of complexity to a simple two-parameter equivalent: the X, Y theorem transforms one elementary three parameter system to another three parameter system. Alternatively stated, Norton's theorem is not concerned in any way with a third node: the X, Y theorem however demonstrates that a third node – T_{rs} – transforms to the T_{ra} node; they are alternatives and cannot appear simultaneously in any circuit formulation.

The X, Y theorem can also be demonstrated by dispensing with the temperature source at T_{av} and linking it instead to ambient T_o (at zero) by the ventilation conductance V and further linking T_1 to T_o via the fabric conduction conductance F_1, as is illustrated in figure 8. This approach is algebraically more laborious.

(b) *The single surface enclosure*

Figure 8a illustrates the thermal circuit for the most elementary of enclosures, one whose entire surface is at the uniform temperature T_1, which is excited by a radiant source Q_r and loses heat to ambient by the ventilation and conduction mechanisms V and F_1. According to the X, Y theorem, it can be replaced by the circuit of figure 8b. It will be seen that the link between T_{ra} and T_1 consists of the radiative and convective mechanisms S_1 and C_1 in parallel to each other, and that the heat flow to the surface is driven by the index temperature T_{ra}, where

$$T_{ra} = (S_1 T_{rs} + C_1 T_{av})/(S_1 + C_1)$$

and is a weighted mean of the radiant star and air temperatures. It will be referred to as the 'rad-air' temperature.

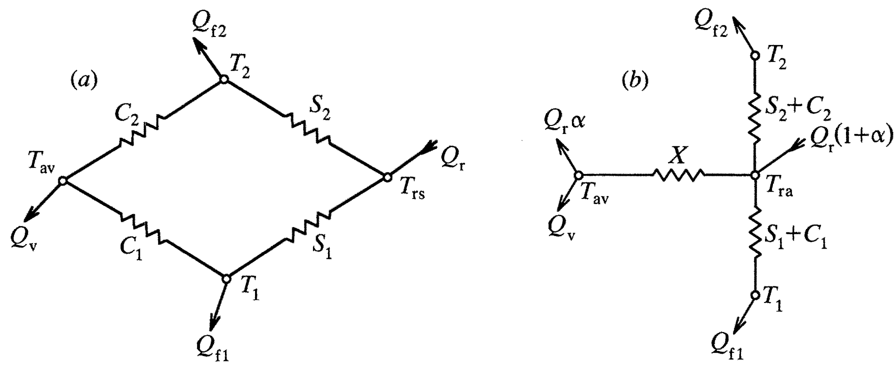

Figure 9. (*a*) Heat exchange in a two-surfaced enclosure, and (*b*) its X, Y transformation.

This argument, which is exact, thus supports the traditional model of heat flow to part of the envelope of an enclosure: T_{ra} is a rigorously arrived at form for T'_a and S_1+C_1 for $A_{wall}\,h'_r + A_{wall}\,h_c$.

The conductive loss conductance L_1 acting between T_{ra} and T_o via T_1 is given as

$$\frac{1}{L_1} = \frac{1}{S_1+C_1} + \frac{d}{A_1\lambda} + \frac{1}{A_1 h_{co}}$$

and a conventional U value can be defined as $u_1 = L_1/A_1$ or

$$\frac{1}{u_1} = \frac{1}{E_1 h_r + h_{c1}} + \frac{d}{\lambda} + \frac{1}{h_{co}}.$$

The X, Y transformation is concerned with the handling of radiation alone; a heat input of Q_a at T_{av} or an input Q_1 at T_1 are handled in the normal way in either the C, S or X, Y formulations. The ventilation loss V – a convective loss to ambient – can also be included, but the direct loss of radiation to ambient through an open window cannot be included since the point from which it is driven, T_{rs}, is excluded from the X, Y transform. (Thus the transform represents an unsymmetrical handling of the symmetrical circuit of figure 7*a*. Figure 7*a* could have been transformed so as to permit a radiant loss to ambient, but then the ventilation loss would have to have been excluded, and the ventilation loss is much the more important.)

Since the dry resultant temperature T_c is linked to the T_{rs} node, and the T_{rs} node does not appear in the rad-air model, T_c itself cannot appear in the rad-air model. Explicit formulae for heat need and temperatures in terms of T_c are given later.

A heat input of Q_a at T_{av} generates temperatures of

$$T_{av} - T_o = \frac{Q_a(X+L_1)}{VX+XL_1+L_1 V} \quad \text{and} \quad T_{ra} - T_o = \frac{Q_a X}{VX+XL_1+L_1 V}.$$

(*c*) *The multisurface enclosure*

The enclosure discussed above is the simplest possible enclosure that includes radiant and convective heat inputs and losses by conduction and ventilation. To generalize it to include a number of surfaces at different temperatures, we can proceed as follows.

Figure 9*a* shows the internal links in an enclosure in which surfaces at two distinct temperatures can be defined, together with their conduction losses Q_{f1} and Q_{f2}.

It can be shown that in figure 9*a*,

$$T_{rs} = T_{av} + \frac{Q_r(S_1+C_1)(S_2+C_2) - Q_{f1}S_1(S_2+C_2) - Q_{f2}S_2(S_1+C_1)}{S_1C_1S_2 + S_1C_1C_2 + S_1S_2C_2 + C_1S_2C_2}.$$

The essential feature of the X, Y theorem is that a central node T_{ra} should be linked to the surfaces by conductances S_1+C_1 and S_2+C_2, as illustrated in figure 9*b*. It is a matter for further enquiry as to what forms the X conductance and the heat input at T_{ra} should take so that the circuit of figure 9*b* should reproduce the figure 9*a* values for T_1, T_2 and T_{rs}, but we will assume that, following the one-surface results, X should equal $(S+C)C/S$, and that the input should be $Q_r(1+C/S)$, where $S = S_1+S_2$ and $C = C_1+C_2$. The value of T_{ra} follows readily. Now the present value at T_{rs}, T'_{rs}, say, must from the definition of T_{ra} be

$$T'_{rs} = T_{ra}(S+C)/S - T_{av}C/S.$$

It is then found from figure 9*b* that

$$T'_{rs} = T_{av} + \frac{Q_r(S_1+S_2+C_1+C_2) - (Q_{f1}+Q_{f2})(S_1+S_2)}{(S_1+S_2)(C_1+C_2)}.$$

Clearly T'_{rs} differs from T_{rs}. If, however, the values of these conductances are such that $C_1/S_1 = C_2/S_2$ (equal to α say), T_{rs} assumes the same value as T'_{rs}:

$$T_{rs} = T'_{rs} = T_{av} + \frac{Q_r(1+\alpha) - (Q_{f1}+Q_{f2})}{C_1+C_2}.$$

T_1 and T_2 too have the same values in the two formulations if $C_1/S_1 = C_2/S_2$. Thus with the condition that C_1/S_1 should equal C_2/S_2, heat flows expressed in terms of the circuit based on the two nodes T_{av} and T_{rs} can be exactly expressed in terms of the circuit centred on the one node, T_{ra}.

The idea is true for a three-surfaced enclosure. For suppose that $C_1/S_1 = C_2/S_2$; as far as conditions at T_{rs} and in the third branch are concerned, branches 1 and 2 can be superposed to form conductances S_1+S_2 and C_1+C_2 with a loss of $Q_{f1}+Q_{f2}$ from the common node. Provided that C_3/S_3 for its part should equal C_1/S_1 or C_2/S_2, the three-branch circuit can be transformed exactly to a one-star form. Clearly this generalizes to a multisurface enclosure.

(*d*) C_2/S_2 *may not equal* C_1/S_1

C_1 and S_1 are defined respectively as A_1h_{c1} and $A_1[(1-\epsilon_1)/\epsilon_1+\beta_1]^{-1}h_r$. The variation of C_j/S_j from surface to surface thus depends on the variation of $h_{cj}/[(1-\epsilon_1)/\epsilon_1+\beta_1]^{-1}$. h_{cj} is taken to have a central value of about 3 W m^{-2} K^{-1} but may be around $\frac{3}{2}$ or $\frac{9}{2}$ W m^{-2} K^{-1} for heated or cooled horizontal surfaces. Building material surfaces have emissivities near to 0.9, though lower values may apply for polished metal surfaces or low emissivity coatings. β tends to 1 and $\frac{1}{2}$ for relatively small and large surfaces respectively. In realistic conditions therefore C_2/S_2 will not exactly equal C_1/S_1, but since the main determining factor for each is area, no gross variation in the ratio from surface to surface is normally to be expected. In view of the evident computational advantages for design purposes of the rad-air formulation, it would appear sensible to define α as

$$\alpha = \frac{C_1+C_2+\dots}{S_1+S_2+\dots} = \frac{C}{S}.$$

(We recall the identity that if

$$\frac{C_1}{S_1} = \frac{C_2}{S_2} = \ldots = \alpha \quad \text{say, then} \quad \alpha = \frac{C_1 + C_2 + \ldots}{S_1 + S_2 + \ldots}.)$$

(*e*) *The structure of the rad-air index*

It has already been remarked that the rad-air temperature T_{ra} is a weighted mean of the radiant and air temperatures in an enclosure. The radiant temperature T_{rs}, however, has a structure, as a result of which we can place two alternative interpretations upon T_{ra}.

In a (radiantly) unheated enclosure, T_{rs} must by continuity be

$$T_{rs}\,(\text{unheated}) = \frac{T_1 S_1 + T_2 S_2 + \ldots}{S_1 + S_2 + \ldots} = \frac{1}{S} \Sigma T_j S_j.$$

The effect of a radiant input Q_r at T_{rs} is to raise T_{rs} by an amount Q_r/S so we can write

$$T_{rs} = T_{rs}(\text{unheated}) + Q_r/S.$$

It follows that the rad-air temperature

$$T_{ra} = \frac{CT_{av}}{S+C} + \frac{ST_{rs}}{S+C}$$

$$= \underset{\text{(i)}}{\frac{CT_{av}}{S+C}} + \underset{\text{(ii)}}{\frac{ST_{rs}\,(\text{unheated})}{S+C}} + \underset{\text{(iii)}}{\frac{Q_r}{S+C}}.$$

In the form, $T_{ra} = \text{(i)} + [\text{(ii)} + \text{(iii)}]$ we have the interpretation already mentioned, namely that T_{ra} is a mix of air and radiant temperatures, but the two components of the radiant contribution are now made explicit.

In the form, $T_{ra} = [\text{(i)} + \text{(ii)}] + \text{(iii)}$, T_{ra} is structurally similar to the well-known 'sol-air' temperature, due to Mackay and Wright nearly 50 years ago. Sol-air temperature is an index temperature set up to handle solar gain at the outside of a wall or roof. It is defined as

$$T_{sa} = T_o + \frac{Q_{sol}}{C_o},$$

where Q_{sol} is the absorbed solar gain on an area A, and C_o is the corresponding convective coefficient between the area and ambient temperature T_o. We have the following correspondences.

1. The present Q_r (long-wave gain) corresponds to Q_{sol} (short-wave gain).
2. $S+C$, the total exchange between the surface and the index temperature, corresponds to C_o.
3. The terms $[\text{(i)}+\text{(ii)}]$ correspond to ambient temperature T_o: if T_{av} and T_{rs} (unheated) happened to be equal (to T_i say), the terms simply reduce to T_i, the direct equivalent of T_o.

The term 'sol' in sol-air refers to just one component: the absorbed solar gain. The term 'rad' in rad-air refers to two components: the radiant field due to the cool enclosure surfaces, and the contribution due to radiation from an internal hot body source.

Finally, it may be noted that dry resultant temperature T_c too depends upon the three components, air temperature, the cool surfaces and any radiant source. It is the high impedance measure of temperature in an enclosure and is the logical correlate with perceptions of thermal comfort. Rad-air temperature is the low impedance measure of temperature in an enclosure and is needed to handle heat gains and losses.

(*f*) *The purpose of thermal models*

The purpose of a model is to relate the various heat flows and temperatures.

Estimates of these quantities can only be as accurate as the assumed characteristics of the enclosure itself, as reflected in the values chosen for its conductances and in what detail its means of excitation are presented. Furthermore, the model is inherently incapable of estimating variation in a quantity for which a lumped value has already been assumed: air temperature may vary by several degrees from floor to ceiling, but a model which works in terms of the space averaged air temperature T_{av} cannot without further consideration provide information about the variation.

In the present context, models may be used in either of two ways. (i) For the design of heating and cooling systems for buildings, the designer will normally select some temperature, probably dry resultant temperature, and find the heat input needed to sustain it. (ii) In the case of possible overheating, the designer 'imposes' a heat input, probably resulting from solar gain, and estimates the daily temperature history in the room. In either case, it may be useful to estimate consequent temperatures and heat flows, and clearly a model adequate for the purpose must be selected.

The models considered here lead to manual methods and are developments of traditional desk methods of design. They are simple to use and therefore can easily be applied for routine design problems. The needs of complicated enclosures should be investigated by using computer models that are capable of processing many parameters.

The relations between heat needs and the various measures of enclosure temperature as a function of choice of comfort temperature are treated in the next section by using the rad-air model.

5. Calculation of enclosure heat needs

The calculation of the heating or cooling load for a room in a building is normally found on the basis of the following information.

1. The client specifies the level of temperature T_c (dry resultant or comfort temperature) to be established for comfort or other purposes (e.g. 20 °C).

2. Climatic data provide a value (the 'design' value) for the ambient temperature T_0 below which the daily mean value falls on some acceptably low proportion of the year (e.g. 5%). This is typically −2 °C for the U.K. but is lower in Northern Europe.

3. From the room dimensions, assumed ventilation rate and details of construction, the services engineer computes the heat losses to ambient by conduction through the building fabric and by air infiltration which results if the required indoor temperature is to be maintained.

In this section, expressions are given for the heat need based on the rad-air formulation of the last section. They parallel those given in the current CIBSE Guide (1986), but are more succinct and logically based. They are expected to serve for the general run of moderate sized rooms. Detailed computer programs should be used for complicated enclosures, e.g. concert halls, atria, where the coarse assumptions regarding air movement in particular, necessarily made for simple methods, may not be adequate.

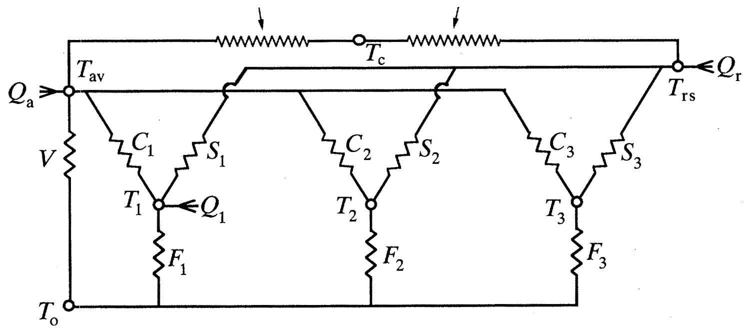

Figure 10. The two-star model for an enclosure. Heat can be input at T_{av}, T_{rs} or at any surface node, e.g. at T_1.

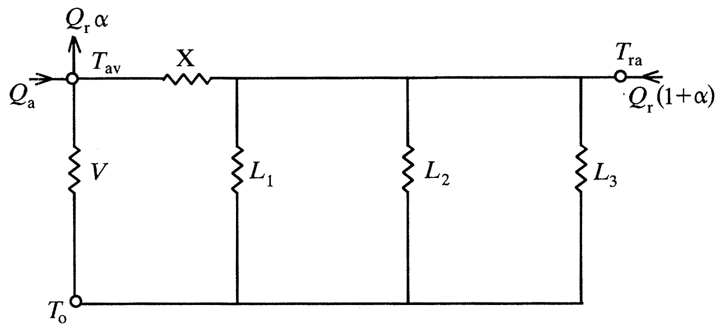

Figure 11. The single or rad-air model for an enclosure. Heat can be input at T_{av} and at T_1 say, but the radiant input is split into an augmented input at T_{ra} and withdrawal of the excess at T_{av}.

(*a*) *Heat inputs*

Heat can be input to a room either by some sort of internal source such as a radiator or at one of the surfaces itself, most probably the floor, by electric resistance wiring or by hot-water piping.

The total output Q_t from an internal source will be split into convective and radiant fractions as

$$Q_a = (1-p)\,Q_t \quad \text{and} \quad Q_r = pQ_t.$$

(*b*) *The two-star model for enclosure heat flows*

Figure 8*a* showed the thermal circuit for the two-star model for an enclosure whose entire surface was at the temperature T_1. Figure 10 illustrates how it can be drawn to include several surfaces. Explicit expressions are given in Davies (1990*b*) for Q_a and Q_r in terms of T_c and the difference $T_{av}-T_{rs}$. They can be evaluated straightforwardly but do not lead to compact algebraic forms and will not be discussed further here.

(*c*) *The rad-air model for enclosure heat flows*

Figure 11 shows the corresponding single-star or rad-air model for an enclosure with several surfaces.

It will be recalled from §4*c* that L_1 denotes the conductance between T_{ra} and T_o via T_1. It is clear, however, that in an enclosure with several surfaces, L_1, L_2, etc., are

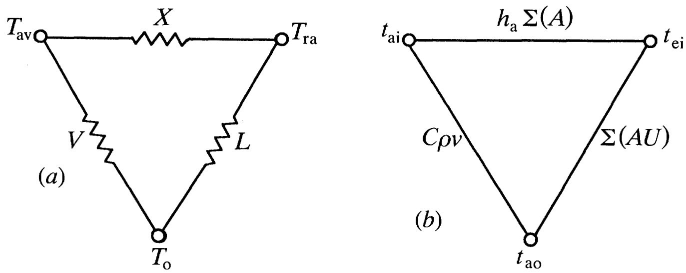

Figure 12. (*a*) Figure 11 redrawn to merge the conductive conductances. (*b*) 'Fig. A5.1 basic conductance network'.

disposed in parallel to each other, and so can be lumped into the single conduction loss conductance and we write

$$L = \Sigma L_j \text{ summing over all surfaces.}$$

Figure 11 can be redrawn as figure 12*a* so as to have an appearance similar to that of fig. A 5.1 in CIBS (1979) and CIBSE (1986) and shown here as figure 12*b*. T_{av}, V and T_o correspond exactly to t_{ai}, $C\rho v$ and t_{ao} respectively. L and $\Sigma(AU)$ only differ in regard to their inner film components (the resistances of the solid elements and of the outer film are handled identically). T_{ra}, however, differs fundamentally from t_{ei} and X similarly differs from $h_a\,\Sigma(A)$.

In the present formulation, four parameters serve to describe the thermal characteristics of the enclosure itself: L, V, X and α; one further parameter, p, is needed if the room is heated by an internal heat source; dry resultant temperature T_c must be specified. From this information, the total heat need itself and certain other temperatures can be estimated as is shown below.

(i) *Heat input from an internal source*

The heat inputs corresponding to an internal heat source consist of: (i) an input of Q_a at T_{av}, (ii) an input of $Q_r\,(1+\alpha)$ at T_{ra}, and (iii) a withdrawal of Q_r, α from T_{av}. Heat balance at these nodes thus leads to a series of expressions, first reported in Davies (1991).

The total heat need is

$$Q_t = \frac{VL+LX+XV}{X+\frac{1}{2}L(1-\alpha)+p[\frac{1}{2}V(1+\alpha)^2-\frac{1}{2}L(1-\alpha^2)]}(T_c-T_o).$$

A number of functions of temperature also follow: the average air temperature,

$$T_{av}-T_o = \frac{X+L-pL(1+\alpha)}{X+\frac{1}{2}L(1-\alpha)+p[\frac{1}{2}V(1+\alpha)^2-\frac{1}{2}L(1-\alpha^2)]}(T_c-T_o);$$

the radiant star temperature,

$$T_{rs}-T_o = \frac{X-L\alpha+p[V(1+\alpha)^2+L\alpha(1+\alpha)]}{X+\frac{1}{2}L(1-\alpha)+p[\frac{1}{2}V(1+\alpha)^2-\frac{1}{2}L(1-\alpha^2)]}(T_c-T_o);$$

the difference between air and radiant star temperatures,

$$T_{av}-T_{rs} = \frac{L(1+\alpha)-p[(L+V)(1+\alpha)^2]}{X+\frac{1}{2}L(1-\alpha)+p[\frac{1}{2}V(1+\alpha)^2-\frac{1}{2}L(1-\alpha^2)]}(T_c-T_o);$$

finally, the rad-air temperature,

$$T_{ra}-T_o=\frac{X+pV(1+\alpha)}{X+\frac{1}{2}L(1-\alpha)+p[\frac{1}{2}V(1+\alpha)^2-\frac{1}{2}L(1-\alpha^2)]}(T_c-T_o).$$

This last expression shows the relationship between the low impedance version T_{ra} and the high impedance version T_c of room temperature.

Some features of these equations may be noted.

1. The total heat need Q_t decreases as we move to a more radiative system (i.e. p increases) if the ventilation loss V exceeds $L(1-\alpha)/(1+\alpha)$. A result of this kind is to be expected on physical grounds.

2. Similarly T_{av} must decrease and T_{rs} must increase as p increases.

3. The equations are inter-related. Thus the difference $T_{av}-T_{rs}$ can be formed from two of the other equations. Further, the total heat need can be expressed as

$$Q_t = V(T_{av}-T_o)+L(T_{ra}-T_o)$$

and this leads to the value given above.

4. X proves to be numerically a large conductance. If its value is taken as infinite, the heat need becomes $Q_t = (V+L)(T_c-T_o)$, and the various measures of room temperature become identical: $T_{av} = T_{rs} = T_{ra} = T_c$. These are the properties of the traditional building model based on the 'air temperature' T'_a (discussed in §1), and used up to the 1960s before the introduction of environmental temperature. The model is still used and would appear to be quite satisfactory for a room that is well insulated and has low ventilation losses.

(ii) *Heat in at a surface*

If Q_a and Q_r are zero, but a heat input Q_1 acts at T_1 – the floor say – it can be shown that

$$Q_1=\frac{F_1}{L_1}\frac{(VX+XL+LV)}{(X+\frac{1}{2}V(1+\alpha))}(T_c-T_o)$$

and

$$T_{av}-T_{rs}=\frac{(1+\alpha)(-V)}{X+\frac{1}{2}V(1+\alpha)}(T_c-T_o).$$

Clearly, a heated floor must lead to a 'cool air' comfort condition. The floor temperature is

$$T_1-T_o=\frac{(VX+XL+LV)(L_1^{-1}-F_1^{-1})+(X+V)L_1/F_1}{X+\frac{1}{2}V(1+\alpha)}(T_c-T_o).$$

A numerical example is given in Appendix C.

6. Discussion

It may be that there is no appreciable difference between the air and radiant temperatures in a room, and if so, the designer can take both ventilation and conduction losses to be driven by a common room temperature, which serves as comfort temperature. This was the only model available up to the 1960s. The environmental temperature model, a single-star model, was developed in the 1960s to handle cases where such an assumption was inadequate. The fundamentals of this model, however, are flawed and the present article has shown how a single-star model, the rad-air model, can be set up in a fully logical manner to serve in its stead. It involves several approximations, however, which should be summarized.

1. The surface to surface radiant exchange cannot be exactly modelled by a surface to star point approximation. The transformation, however, if carried out by the least-squares procedure outlined in §2, is quite good as table 1 illustrates.

2. In applications, the designer is likely to use, not 'original' β values, but β values found by using the regression equation for β_j and there is a standard deviation of 0.0068 between these values. The deterioration in fit is only marginal. (A default value of $\beta = \frac{5}{6}$, the exact value for a cube, might prove acceptable for all surfaces.)

3. A further assumption that has to be made is that the radiant star temperature T_{rs} serves as an adequate estimate of the space-averaged observable radiant temperature T_{rv}. This assumption is not so satisfactory since T_{rv} varies according to the position of the source and T_{rs} may lie outside of its range. Against this, it has to be recognized that the local radiant temperature T_{rp} must itself vary in the occupiable space from high values close to the source to low values remote from it and that its value depends upon details of the sensor itself.

4. Lastly, the development of the rad-air model involved the assumption that the ratio C_j/S_j was the same for every surface. As mentioned earlier, this is likely to be near enough true in many cases, but low emissivities or high convection coefficients on large surfaces might make the assumption inappropriate. The two-star model might then be better suited.

In restricted circumstances, rad-air and environmental temperatures have the same form. They are (i) that the enclosure be of cubic form so that $\beta_j = \frac{5}{6}$ for all surfaces, (ii) that all its surfaces be black-body so that $\epsilon_j = 1$ and the Guide's $E = 1$, (iii) that all surfaces have the same convective coefficient h_c, and (iv) that there is no radiative input Q_r to the enclosure. In this case, the two measures are related in an identical manner to the remaining parameters:

$$T_{ei} \equiv T_{ra} = (\tfrac{6}{5}h_r T_m + h_c T_{av})/(\tfrac{6}{5}h_r + h_c),$$

where T_m is the area-weighted mean surface temperature. (This result is independent of the conductive losses F_j and the ventilation loss V from the enclosure to ambient.) In normal enclosures with high emissivity surfaces and moderate air speeds the first three of these conditions are met approximately. Thus heat needs computed as suggested by the numerical scheme of the 1986 CIBSE Guide are not likely to be in error. Indeed, the Guide's scheme is justified by the rad-air model rather than by the reasoning the Guide itself advances.

This article has summarized several features believed to be new to building heat transfer.

1. The well-established radiosity node transforms under linearization of differences of fourth powers of temperature to the 'black-body equivalent node', T_{jb}. Long-wave radiation from an internal source upon a surface can be treated as though fully absorbed either at the radiosity node or the black-body node, and indeed solar radiation can be handled similarly. This proves to be a simpler approach than summing multiple reflections.

2. The radiant star network can be optimally designed by using least-squares methods.

3. It is necessary to have two separate measures for the global radiant temperature

in an enclosure: T_{rs} and T_{rv}. T_{rs} is physically meaningless but is easily calculated; T_{rv} is physically significant and is needed in connection with thermal comfort.

4. The justification for merging the convective and radiative flow from the room as a whole to a surface is based on a seemingly new theorem (§4*a*) with its extension to two or more surfaces. The rad-air temperature which results from the argument is the indoor equivalent of the well-established 'sol-air' temperature.

5. Most of the links in a thermal circuit are 'high conductance' links and the associated nodes are high conductance nodes. It has now become apparent however that dry resultant temperature must be modelled as a low conductance (high impedance) node.

It is to be hoped that recognition of these innovations and developments will serve to remove the sloppiness that has so long bedevilled design aspects of building heat transfer and that they may afford it the rigour that is normally to be found in applied science disciplines.

Appendix A. Point of input of radiant energy

The problem of radiation from some source falling upon a surface is usually handled by considering the absorbed and reflected fractions at the surface. We have to show that it can equally be treated as though it is input at the radiosity node. Following the usual approach, consider two large parallel planes of emissivities ϵ_1 and ϵ_2 and at absolute zero, so that their emittances are zero. Suppose that a particle of radiant energy H falls on surface 1. Since emissivity and absorptivity are equal, the absorbed flux is $H\epsilon_1$. The remainder, $H(1-\epsilon_1)$, is reflected to surface 2 and $H(1-\epsilon_1)\,\epsilon_2$ is absorbed there. The pattern of absorption at the surfaces following successive reflections is shown:

energy absorbed at surface 1	energy absorbed at surface 2
$H\epsilon_1$,	$H(1-\epsilon_1)\,\epsilon_2$,
$H(1-\epsilon_1)(1-\epsilon_2)\,\epsilon_1$,	$H(1-\epsilon_1)(1-\epsilon_2)(1-\epsilon_1)\,\epsilon_2$,
$H(1-\epsilon_1)(1-\epsilon_2)(1-\epsilon_1)(1-\epsilon_2)\,\epsilon_1, \ldots$	$H(1-\epsilon_1)(1-\epsilon_2)(1-\epsilon_1)(1-\epsilon_2)(1-\epsilon_1)\,\epsilon_2, \ldots.$

On summing the infinite geometrical series, the total fluxes are

$$H_1 = H\epsilon_1/(\epsilon_1+\epsilon_2-\epsilon_1\,\epsilon_2), \quad H_2 = H(\epsilon_2-\epsilon_1\,\epsilon_2)/(\epsilon_1+\epsilon_2-\epsilon_1\,\epsilon_2).$$

According to the Oppenheim formulation, the circuit for this situation is as shown in figure 13.

The conductance between the radiosity nodes is simply A since $F_{12} = 1$. Elementary circuit analysis readily shows that $H_1 = H\epsilon_1/(\epsilon_1+\epsilon_2-\epsilon_1\,\epsilon_2)$, as did the reflections method.

Clearly, in a multisurface enclosure the handling of the reflections becomes complicated while the consequences of inputting radiation at one or more radiosity nodes or black-body equivalent nodes can be found by routine circuit analysis.

Appendix B. Calculation of an optimal radiant star circuit

A procedure was sketched in §2 to find the optimal star configuration to represent radiant exchange in a rectangular enclosure. As an illustration, we consider the cuboid where $l = 1.5849$, $d = 0.6310$, and $h = 1.0000$. Only the ratios of these lengths

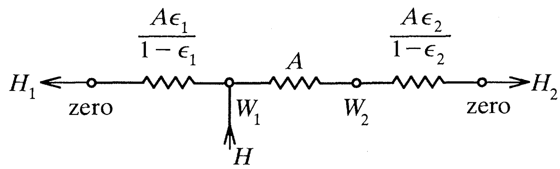

Figure 13. The thermal circuit for radiant exchange between two infinite parallel plates.

Table 5. *View factors between surfaces*

	north	east	floor	south	west	ceiling
north	—	0.112	0.185	0.406	0.112	0.185
east	0.282	—	0.184	0.282	0.068	0.184
floor	0.293	0.116	—	0.293	0.116	0.183
south	0.406	0.112	0.185	—	0.112	0.185
west	0.282	0.068	0.184	0.282	—	0.184
ceiling	0.293	0.116	0.183	0.293	0.116	—

Table 6. *Conductance between surface* (m^2)

	north	east	floor	south	west	ceiling
north	1.585	−0.178	−0.293	−0.643	−0.178	−0.293
east		0.631	−0.116	−0.178	−0.043	−0.116
floor			1.000	−0.293	−0.116	−0.183
south				1.585	−0.178	−0.293
west					0.631	−0.116
ceiling						1.000

are relevant: the cuboid could be taken to be a room of dimensions for example, 3.96 m × 1.58 m × 2.5 m in height. The $l \times h$ walls form the north and south walls, the $d \times h$ walls the east and west walls and the $l \times d$ surfaces the floor and ceiling.

The array of view factors F_{jk} for exchange from the surface in the column to the surface in the row is as listed in table 5. There are nine numerically distinct values here. (It is a matter of chance that three of them are nearly the same.) For reasons of symmetry,

$$F(\text{north}, \text{east}) = F(\text{north}, \text{west}) = F(\text{south}, \text{east}) = F(\text{south}, \text{west})$$

and that

$$F(\text{north}, \text{south}) = F(\text{south}, \text{north}), \text{ etc.}$$

The sum of all view factors on a row is unity.

The conductance between the north and east walls for example is $(1.5849 \times 1.0000) \times 0.112 = 0.178\ \text{m}^2$. There are six such numerically distinct values.

In the matrix equation $\boldsymbol{GW} = \boldsymbol{Q}$, the $\boldsymbol{G}$ matrix is as given in table 6. The matrix is symmetrical. The sum of the off-diagonal elements in any row or column is the negative of the diagonal element. (Generally speaking, the diagonal element in a matrix of this kind exceeds this sum because it includes links external to the system, but in the present case, there are no additional conductances providing any external connections to the array of radiant conductances.)

Table 7. *Net resistances between surfaces* (m^{-2})

	north	east	floor	south	west	ceiling
north	—	1.924	1.336	0.898	1.924	1.336
east		—	2.268	1.924	2.968	2.268
floor			—	1.336	2.268	1.690
south				—	1.924	1.336
west					—	2.268
ceiling						—

As explained, the matrix inverse to the first five rows and columns of $\boldsymbol{G}$ has to be found, from which the exact net resistance R^{Δ}_{jk} (units m^{-2}) between any two nodes is calculated (see table 7). The matrix is symmetrical and again,

$$R^{\Delta}(\text{north, east}) = R^{\Delta}(\text{north, west}) = R^{\Delta}(\text{south, east}) = R^{\Delta}(\text{south, west}).$$

The north and south walls provide the largest areas in the room and the resistance between them, 0.898 m^{-2}, is accordingly the smallest in the array. The east and west walls constitute the smallest surfaces and have the largest resistance.

These exact resistances are used to compute the approximate resistances R^{*}_{jk} of the star-based system. Starting with guessed values for the β_js, the sum of products is formed:

$$S_p = \sum_j \sum_k (G^{\Delta}_{jk} - G^{*}_{jk})(R^{*}_{jk} - R^{\Delta}_{jk}),$$

where $G^{\Delta}_{jk} = 1/R^{\Delta}_{jk}$ and $G^{*}_{jk} = 1/R^{*}_{jk}$. S_p can be minimized. A numerical procedure to do this is based on the idea that near its minimum, S_p can be expressed approximately as a parabaloid. If x denotes β_1/A_1, etc.,

$$S_p = ax^2 + by^2 + cz^2 + dxy + eyz + fzx + gx + hy + jz + k,$$

where the coefficients are functions of the delta resistances and the current set of star resistances.

With the assumed initial values for the βs, that is, for the point (x, y, z), the value for S_p can be found, together with the three values of its first differential coefficient, the three values of type $\partial^2 S_p/\partial x^2$ and the three terms of type $\partial^2 S_p/\partial x\,\partial y$. In the parabolic approximation, $\partial^2 S_p/\partial x\,\partial y$ for example is equal to d; the coefficients a to f can be found in this way, and knowing the numerical values of the first differential coefficients, we can find the further coefficients g, h and j.

Now the minimum of the parabola, at the point (x', y', z'), is determined from relations of type $\partial S_p/\partial x = 0$ or $2ax' + dy' + fz' = -g$, and two similar equations, solution of which gives (x', y', z'). (x', y', z') provides a better set of β values than did the initial choice at (x, y, z). The process is continued until a solution of stated accuracy is reached.

The NAG Library routine E04EAA based on this process was applied to a series of enclosures as listed in §2. For the enclosure discussed above,

$$\beta(\text{north}) = 0.730, \quad \beta(\text{east}) = 0.915 \quad \text{and} \quad \beta(\text{floor}) = 0.856.$$

The largest surface has the smallest β value and conversely. The minimum value of S_p was 0.003513. This is formed as the sum of 15 positive products so the quantity $\delta_{60} = \surd(\frac{1}{15}S_p)$ can be interpreted as the 'root mean square fractional difference in

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resistance (or conductance, it matters little which) between the star and delta networks'. Here, δ_{6o} in percent is equal to 1.53 and is entered in this form in table 1.

Computations on the complete set of enclosures provided the regression equation for β given in the main text and the values so estimated in the present case were $\beta(\text{north}) = 0.740$, $\beta(\text{east}) = 0.923$ and $\beta(\text{floor}) = 0.856$. These are of course not optimal and so will lead to a somewhat larger value of S_p than that leading to 1.53. The corresponding value of δ, δ_{6e}, was 1.72. See table 1.

The actual star resistance between say the north and south walls might be greater or less than the exact value. The value of $(R^*_{jk}/R^{\Delta}_{jk} - 1)$ indicates the fractional error in the star resistance between surfaces j and k. Its values in the current case, with R^*_{jk} computed from regression equation β values, are listed:

north–south	east–west	floor–ceiling	north–east	east–floor	floor–north
0.0408	−0.0142	0.0148	0.0033	0.0231	−0.0084

The largest and smallest of these errors are noted in table 1 as percentages.

Appendix C. A worked example

As an example of use of the rad-air model, the conductive and ventilation losses of a test enclosure will be calculated; values for comfort and ambient temperatures T_c and T_o will be specified and so the total heat need and values for the various enclosure temperatures will be found. (It follows the example given in Davies (1991).)

Consider an enclosure $4 \times 3 \times 2\frac{1}{2}$ m^3 with three vertical walls which for the purposes of the present plant sizing can be taken as adiabatic. There is a single-glazed window in the outer ($4 \times 2\frac{1}{2}$ m^2) wall through which there will be a sizable conduction loss but the window will be assumed to be coated to give it a low emissivity. Further but smaller losses will be assumed through the remainder of the outer wall and through the ceiling and the floor leading to a value for the conductive transmittance f_j. (f_j denotes the transmittance between a wall interior surface and ambient. It depends upon the thicknesses and conductivities of the wall layers, together with the film coefficient of the outer film but not of course upon the internal radiative and convective processes. It is sufficient here to assume a value for f_j without specifying the construction that leads to it.) Normal values of emissivity at all surfaces other than the window will be assumed, and appropriate convective coefficients will be selected. A ventilation loss will be assumed. Each surface will be taken to be isothermal.

A ventilation rate of two air changes per hour will be taken. Table 8 provides values for the surface parameters. They are based on expressions given earlier. So

$$\alpha = \Sigma C_j / \Sigma S_j = 174.0/345.52 = 0.5036,$$

$$X = (1+\alpha)\, C_j = 1.5036 \times 174.0 = 261.6 \text{ W K}^{-1},$$

$$L = 35.75 \text{ W K}^{-1},$$

$$V = 1200 \times (2/3600) \times (4.0 \times 3.0 \times 2.5) = 20.0 \text{ W K}^{-1}.$$

This information summarizes the heat transfer characteristics of the enclosure. The procedure has been illustrated here to show the flexibility in handling different convective coefficients, emissivities and enclosure geometries. In routine use,

Table 8. *Room surface parameters*

surface	dimensions	area, A_j/m^2	outward conduction loss, f_j ($W\ m^{-2}\ K^{-1}$)	convective exchange: h_{cj} ($W\ m^{-2}\ K^{-1}$)	convective exchange: C_j ($W\ K^{-1}$)	radiative exchange: ϵ_j	radiative exchange: β_j	radiative exchange: E_j	radiative exchange: S_j ($W\ K^{-1}$)	overall transfer: u_j ($W\ m^{-2}\ K^{-1}$)	overall transfer: L_j ($W\ K^{-1}$)
floor	4.0 × 3.0	12.0	0.8	2.0	24.0	0.9	0.7957	1.1028	75.43	0.730	8.75
ceiling	4.0 × 3.0	12.0	0.7	3.5	42.0	0.9	0.7957	1.1028	75.43	0.653	7.84
side wall	3.0 × 2.5	7.5	0	3.0	22.5	0.9	0.8904	0.9985	42.69	0	0
side wall	3.0 × 2.5	7.5	0	3.0	22.5	0.9	0.8904	0.9985	42.69	0	0
rear wall	2.5 × 4.0	10.0	0	3.0	30.0	0.9	0.8392	1.0523	59.98	0	0
outer wall	—	7.0	0.7	3.0	21.0	0.9	0.8392	1.0523	41.99	0.649	4.55
window	—	3.0	20.0	4.0	12.0	0.4	0.8392	0.4275	7.31	4.870	14.61
		59.0			174.0				345.52		35.75

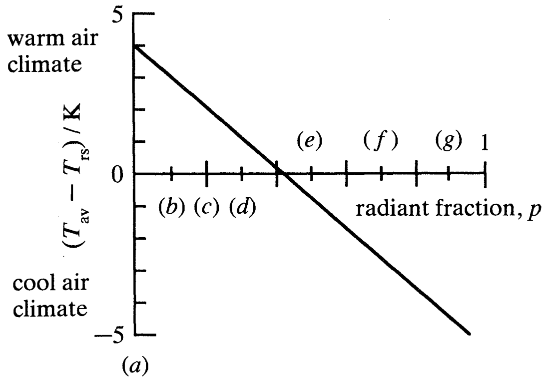

Figure 14. Variation of $T_{av} - T_{rs}$ with form of heating system for the worked example. (*a*) Forced warm air heaters; (*b*) natural convectors and convector radiators; (*c*) multicolumn radiators; (*d*) double and treble panel radiators and double column radiators; (*e*) single column radiators, block storage heaters; (*f*) vertical heaters; (*g*) high temperature radiant systems.

however, such detail may not be needed. The designer might start with conventional values for these quantities: take

$$\alpha = \tfrac{1}{2}, \quad X = 4.5\Sigma A_j, \quad L = \Sigma A_j U_j,$$

using conventional U values with $\beta = \frac{5}{6}$,

$$V = 1200\,(n/3600) \times \text{volume, as above.}$$

If, however, the calculation has to be extended to compute surface temperatures, the correct radiative and convective coefficients for the surface must be used. The inside film transmittance h_j for surface j is given as

$$h_j = \frac{\epsilon_j h_r}{1 - \epsilon_j + \epsilon_j \beta_j} + h_{cj}.$$

With conventional values of $\epsilon_j = 0.9$, $h_r = 5.7\ \mathrm{W\,m^{-2}\,K^{-1}}$, $\beta_j = \frac{5}{6}$ and $h_{cj} = 3\ \mathrm{W\,m^{-2}\,K^{-1}}$, $h_j = 9.03\ \mathrm{W\,m^{-2}\,K^{-1}}$, or a resistance of $0.11\ \mathrm{m^2\,K\,W^{-1}}$. Values of surface resistance given in the 1986 CIBSE Guide, Table A 3.5 range from 0.10 to $0.14\ \mathrm{m^2\,K\,W^{-1}}$.

(*a*) *Hot-body internal source*

With the above values, the total heat need Q_t is given by

$$Q_t = 56.56\,(T_c - T_o)/(1 + 0.034p).$$

Thus the heat need depends a little upon the choice of heating system and for this enclosure decreases very slightly as one progresses towards a more radiant system.

The choice will have a marked effect, however, upon whether the system produces a 'warm air' or a 'cool air' quality of comfort, that is to say upon the value of $(T_{av} - T_{rs})$, which is given as

$$T_{av} - T_{rs} = \frac{0.20 - 0.47p}{1 + 0.034p}(T_c - T_o).$$

Suppose that $T_o = 0$ °C and that the dry resultant temperature T_c is to be 20 °C. Figure 14 shows the value of $(T_{av} - T_{rs})$ for various means of heat input.

Suppose that the heating system to be chosen is a double column radiator, centrally placed, so that the whole of its radiant output is useful in heating the space. (This avoids the complication of separate consideration of the radiation leaving the back of the radiator if it had been placed against a wall.) According to the 1986 CIBSE Guide, §A9, the fraction of heat emitted radiantly is about 0.3, so we take $p = 0.3$.

It follows from expressions in §5 that the total heat input is 1119.8 W, of which 784.2 W is input convectively and 335.6 W radiatively. 411.6 W is lost by ventilation and 708.2 W by conduction through the fabric. (These values are quoted to four significant figures for computational purposes only. It is recognized that in practice such estimates may well be uncertain by perhaps 10%.)

The average air temperature T_{av} is 20.58 °C, the radiant star temperature $T_{rs} =$ 19.42 °C, $T_{av} - T_{rs} = 1.16$ K, and the rad-air temperature $T_{ra} = 19.81$ °C.

Thus there is little variation between the various measures of temperature in this case, and in particular the two global measures, T_{ra} and T_c are very close.

(*b*) *Floor heating*

According to the equations in §5, the heat input to the floor to maintain $T_c = 20$ °C is 1213.5 W, and the difference $T_{av} - T_{rs} = -2.17$ K. Rather more heat is needed than was the case for an internal source of heat.

The temperature of the floor is calculated to be 29.75 °C. Evidently, underfloor heating would be unsuitable in this case.

Solar gains absorbed at the floor are to be handled similarly.

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